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ALGEBRAIC COMPACTNESS OF ULTRAPOWERS AND REPRESENTATION TYPE

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It is shown that for certain clases of Artin algebras infinite representation type is equivalent to the existence of a module M all of whose ultrapowers $M^{\mathbb{N}}/\mathscr{F}$ fail to be algebraically compact.

1. Introduction. As is well known, an Artin algebra R is of finite representation type, i.e., R admits only finitely many isomorphism classes of indecomposable right or left modules, if and only if R has right pure global dimension zero (combine [**1**] with [**5**] or [**13**]). The latter condition means that all right R-modules are pure-injective or, equivalently, algebraically compact; in particular, it of course entails algebraic compactness of all non-trivial countable ultraproducts of right R-modules. (Call an ultraproduct countable if it extends over a countable index set, non-trivial if the corresponding ultrafilter is non-principal.)

Let us leave this "extreme" case. How do the non-trivial countable ultraproducts of R-modules reflect higher pure global dimensions of the ground ring?

The situation where R is countable is exceptional on that score: namely, non-trivial countable ultraproducts of R-modules are *always* algebraically compact in that case (a short proof can be found in the appendix; see also [**4**, Theorem 42.1] and [**6**, Remarque 7.12]). On the other hand, if R is an uncountable Artin algebra of a pure global dimension exceeding 0, then "usually" there exist non-trivial countable ultraproducts of finitely generated modules which fail to be algebraically compact (see [**8**] for precise statements). Interestingly, this conclusion can be translated back to a stronger condition on the pure global dimension of R; namely, it rules out the values 0 *and* 1.

The picture for ultra*powers* is quite different in that arbitrary ultrapowers of finitely generated modules over an Artin algebra are algebraically compact (see the appendix for details). Any case of failure of algebraic compactness for a non-trivial countable ultrapower $M^{\mathbb{N}}/\mathscr{F}$ thus involves a non-finitely generated module M over an uncountable Artin algebra of infinite type. In fact, for Artin algebras R