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HYPERBOLICITY OF SURFACES MODULO RATIONAL AND ELLIPTIC CURVES

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Let X be a smooth compact complex surface of general type and let D be the union of all rational and elliptic curves in X. If there exist a complex torus T of dimension ≥ 2 and a nontrivial holomorphic map $X \to T$ whose image contains no elliptic curves then X is hyperbolic modulo D. In particular, if X has irregularity $h^0(X, \Omega_X^1) \geq 2$ and its Albanese variety is not isogenous to a product of elliptic curves then X is hyperbolic modulo D.

Introduction. A complex space X is called *hyperbolic* if the Kobayashi pseudo-distance d_X on X is a distance, i.e. if $d_X(x, x') > 0$ for $x \neq x'$ **[K]**. If D is a subset of X and $d_X(x, x') > 0$ unless $x = x'$ or $x, x' \in D$, we say that X is *hyperbolic modulo D*. Let X be a surface of general type. M. Green has made the following conjectures:

Conjecture A. The image of every nonconstant holomorphic map $\mathbf{C} \to X$ lies in a rational or elliptic curve in X, and

Conjecture B. X is hyperbolic modulo the union of all its rational and elliptic curves.

Conjecture A is known to be true for surfaces with irregularity $h^0(X, \Omega_X^1) > 2$ **[GG, OC]** and surfaces with irregularity 2 and simple Albanese variety **[G]**. We use these facts together with Brody's theorem (1.2 below) to prove the following:

THEOREM. *Let X be a smooth compact complex surface of general type and let D be the union of all rational and elliptic curves in X. If there exist a complex torus T of dimension ≥ 2 and a nontrivial holomorphic map $X \to T$ whose image contains no elliptic curves then X is hyperbolic modulo D.*

COROLLARY. *If X is a smooth compact complex surface of general type with irregularity ≥ 2 whose Albanese variety is not isogenous to a product of elliptic curves then X is hyperbolic modulo its rational and elliptic curves.*