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WEIGHTS INDUCED BY HOMOGENEOUS POLYNOMIALS

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Let B be the unit ball and S the unit sphere in $\mathbb{C}^n$ $(n \geq 2)$. Let σ be the unique normalized rotation-invariant Borel measure on S and m the normalized area measure on $\mathbb{C}$.

We first prove that if Λ is a holomorphic homogeneous polynomial on $\mathbb{C}^n$ normalized so that Λ maps B onto the unit disk U in $\mathbb{C}$ and if $\mu = \sigma[(\Lambda|_S)^{-1}]$, then $\mu \ll m$ and the Radon-Nikodym derivative $d\mu/dm$ is radial and positive on U. Then we obtain the asymptotic behavior of $d\mu/dm$ for a certain, but not small, class of functions Λ. These results generalize two recent special cases of P. Ahern and P. Russo. As an immediate consequence we enlarge the class of functions for which Ahern-Rudin's Paley-type gap theorems hold.

1. Introduction. Let n be a positive integer. Write $B = B_n$ for the unit ball in $\mathbb{C}^n$ and let $S = S_n = \partial B_n$. When $n = 1$, we use the notation U and T in place of B_1 and S_1, respectively. We shall let $\sigma = \sigma_n$ denote the unique normalized rotation-invariant Borel measure on S and m the normalized area measure on $\mathbb{C}$. The symbol P_n stands for the class of holomorphic homogeneous polynomials Λ on $\mathbb{C}^n$ normalized so that $\Lambda(B) = U$. The maximum modulus set $\Lambda^{-1}(T) \cap S$ of $\Lambda \in P_n$ is denoted by $\operatorname{Max}\Lambda$. It is assumed $n \geq 2$ in the rest of the paper unless otherwise specified.

To begin with, let us look at some special cases which motivated the main results of this paper. We have the following "change-of-variables" formula for $\Lambda \in P_n$ of degree 1 [**Ru**, Section 1.4]:

$$(1) \qquad \int_S \psi \circ \Lambda^* \, d\sigma = (n-1) \int_U \psi(\lambda)(1 - |\lambda|^2)^{n-2} \, dm(\lambda).$$

Here $\Lambda^* = \Lambda|_S$ and ψ denotes an arbitrary nonnegative Borel function on U. The similar integral formula for $\Lambda(z) = z_1^2 + \cdots + z_n^2$ has been recently proved by P. Russo [**Rus**]:

$$(2) \qquad \int_S \psi \circ \Lambda^* \, d\sigma = (n-1)/2 \int_U \psi(\lambda)(1 - |\lambda|^2)^{(n-3)/2} \, dm(\lambda).$$

Also, P. Ahern [**A**] has shown that if $\Lambda(z) = n^{n/2} z_1 \cdots z_n$, then

$$(3) \qquad \int_S \psi \circ \Lambda^* \, d\sigma = \int_U \psi(\lambda) w(\lambda) \, dm(\lambda)$$