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DIFFERENTIABILITY PROPERTIES OF SUBFUNCTIONS FOR SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

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We obtain sharp differentiability results for subfunctions for second order ordinary differential equations $y'' = f(x, y, y')$ on $[a, b]$. In the process we show that a subfunction satisfies a second order differential inequality similar to that satisfied by a lower solution. We show that a subfunction can be used in maximum principle arguments in the same way one uses a lower solution. As an application of these results we give necessary and sufficient conditions on a function in order that there is a differential equation for which it is a subfunction. We use our results together with the Perron method to improve on some existence results for two point boundary value problems obtained by Jackson, using Perron's method.

1. Introduction. Subfunctions and solutions of differential inequalities have been used for a long time to establish existence theorems and properties of solutions for both ordinary and partial differential equations. In 1915, Perron [**11**] used solutions of differential inequalities to establish the existence of a solution of the initial-value problem for the first order equation $y' = f(x, y)$. In 1923 Perron [**12**] used subharmonic functions to study the Dirichlet problem for Laplace's equation for bounded plane domains. Perron used local solvability of the Dirichlet problem for circles and properties of subharmonic functions to prove the existence of a generalized solution which is harmonic in the interior of the domain, allowing the question of whether or not it assumes the specified values at the boundary to be treated separately. The success of subharmonic functions leads to various extensions of the concept and a careful study of the properties of these related functions. One early extension was to second order ordinary differential equations. We consider second order ordinary differential equations of the form

$$y'' = f(x, y, y') \tag{1.1}$$

where $f\colon [a, b] \times \mathbf{R}^2 \to \mathbf{R}$ is continuous.

By a solution of (1.1) on a subinterval I of $[a, b]$, we mean a function $y\colon I \to \mathbf{R}$ which is twice continuously differentiable on I and satisfies