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HARDY INTERPOLATING SEQUENCES OF HYPERPLANES

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A sufficient condition is given on unions of complex hyperplanes in the unit ball of C^n so that they allow extension of functions in the Hardy H^1 space. The result is compared to Varopoulos' theorem about zeros of H^p functions.

1. Notations and definitions. For $z, w \in C^n$,

$$z \cdot \bar{w} = \sum_{i=1}^{n} z_i \bar{w}_i,$$

$$B^n = \{z \in C^n \colon |z|^2 = z \cdot \bar{z} < 1\}.$$

For $a_k \in B^n$, $a_k \neq 0$,

$$a_k^* = \frac{a_k}{|a_k|}.$$

λ_p = p real-dimensional Lebesgue measure. For instance, on C, $-\frac{i}{2} dz \wedge d\bar{z} = d\lambda_2$.

Automorphisms of the ball.

$$\phi_k(z) := \phi_{a_k}(z) := \frac{a_k - P_k(z) - s_k Q_k(z)}{1 - z \cdot \bar{a}_k}$$

where $P_k(z) := \frac{z \cdot \bar{a}_k}{|a_k|^2} a_k$ is the projection onto the complex line through a_k, $Q_k(z) := z - P_k(z)$ is the projection onto the complex hyperplane perpendicular to a_k, $s_k^2 := 1 - |a_k|^2$.

The map ϕ_k is an involution of the ball (see Rudin [4]). Note that

$$Q_k(B^n) = \{z \colon P_k(z) = 0\} = \{z \colon z \cdot \bar{a}_k = 0\}.$$

We write

$$d_G(z, w)^2 := |\phi_w(z)|^2 = 1 - \frac{(1 - |z|^2)(1 - |w|^2)}{|1 - z \cdot \bar{w}|^2}.$$

This is an *invariant* distance: if ϕ is an automorphism of the ball (i.e. any composition of unitary transformations and the above involutions), $d_G(\phi(z), \phi(w)) = d_G(z, w)$.