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ON THE FINEST LEBESGUE TOPOLOGY ON THE SPACE OF ESSENTIALLY BOUNDED MEASURABLE FUNCTIONS

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Let (Ω, Σ, μ) be a σ-finite measure space and let $\mathscr{T}_0$ and $\mathscr{T}_\infty$ denote the usual metrizable topologies on L^0 and L^∞, respectively. In this paper the space L^∞ with the mixed topology $\gamma(\mathscr{T}_\infty, \mathscr{T}_0|_{L^\infty})$ is examined. It is proved that $\gamma(\mathscr{T}_\infty, \mathscr{T}_0|_{L^\infty})$ is the finest Lebesgue topology on L^∞, and that it coincides with the Mackey topology $\tau(L^\infty, L^1)$.

1. Introduction. For notation and terminology concerning Riesz spaces and locally solid topologies we refer to [**1**].

Let (Ω, Σ, μ) be a σ-finite measure space, and let L^0 denote the set of equivalence classes of all real valued μ-measurable functions defined and finite a.e. on Ω. Then L^0 is a super Dedekind complete Riesz space under the ordering $x \le y$, whenever $x(t) \le y(t)$ a.e. on Ω. The Riesz F-norm

$$\|x\|_0 = \int_\Omega |x(t)|(1 + |x(t)|)^{-1} f(t)\, d\mu \quad \text{for } x \in L^0,$$

where a function $f\colon \Omega \to (0, \infty)$ is μ-measurable with $\int_\Omega f(t)\, d\mu = 1$, determines a Lebesgue topology on L^0, which we will denote by $\mathscr{T}_0$ (see [7, I, §6], [**1**, Theorem 24.67]). This topology generates convergence in measure on the measurable subsets of Ω whose measure is finite. We will denote by $\mathscr{T}_\infty$ the topology on L^∞ generated by the usual B-norm

$$\|x\|_\infty = \operatorname*{ess\,sup}_{t \in \Omega} |x(t)|.$$

Moreover, we denote by $\sigma(L^\infty, L^1)$, $\tau(L^\infty, L^1)$ and $\beta(L^\infty, L^1)$ the weak, Mackey and strong topologies on L^∞ respectively, with respect to the dual pair $(L^\infty, L^1, \langle\ ,\ \rangle)$, where

$$\langle x, y\rangle = \int_\Omega x(t) y(t)\, d\mu \quad \text{for } x \in L^\infty, y \in L^1.$$

In this paper we shall examine the space L^∞ with the mixed topology $\gamma(\mathscr{T}_\infty, \mathscr{T}_0|_{L^\infty})$. This topology is defined as follows. Take a sequence