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# ON THE BEHAVIOUR OF CAPILLARIES AT A CORNER

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**Consider the solution of the capillary surface equation over domains with a corner. It is assumed that the corner is bounded by lines. If the corner angle $2\alpha$ satisfies $0 < 2\alpha < \pi$ and $\alpha + \gamma < \pi/2$ where $0 \leq \gamma < \pi/2$ is the contact angle between the surface and the container wall then it is shown that the leading term which was discovered by Concus and Finn is equal to the solution up to $O(r^{\varepsilon})$ for an $\varepsilon > 0$ where $r$ denotes the distance from the corner.**

We consider the non-parametric capillary problem in presence of gravity over a bounded base domain $\Omega \subset \mathbb{R}^2$ with a corner. That means, we seek a surface $S$: $u = u(x)$, defined over $\Omega$, such that $S$ meets vertical cylinder walls over the boundary $\partial\Omega$ in a prescribed constant angle $\gamma$ such that the following equations are satisfied, see Finn [3],

$$\operatorname{div} Tu = \kappa u \quad \text{in } \Omega, \tag{1}$$

$$\nu \cdot Tu = \cos\gamma \quad \text{on the smooth parts of } \partial\Omega, \tag{2}$$

where

$$Tu = \frac{Du}{\sqrt{1+|Du|^2}},$$

$\kappa = \text{const.} > 0$ and $\nu$ is the exterior unit normal on $\partial\Omega$.

Let the origin $x = 0$ be a corner of $\Omega$ with interior angle $2\alpha$ satisfying

$$0 < 2\alpha < \pi. \tag{3}$$

We assume that the corner is bounded by lines near $x = 0$, see Figure 1. Furthermore, we assume that the contact angle satisfies

$$0 \leq \gamma < \frac{\pi}{2}. \tag{4}$$

Concus and Finn [2] have shown that $u$ is bounded near $x = 0$ if and only if $\alpha + \gamma \geq \pi/2$ is satisfied.

In the case $\alpha + \gamma > \pi/2$ there exists an asymptotic expansion of $u$ near the origin, cf. [4]. In the borderline case $\alpha + \gamma = \pi/2$ Tam [5]