# ISOMETRIES OF TRIDIAGONAL ALGEBRAS 

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#### Abstract

Let $\operatorname{Alg} \mathscr{L}$ be a tridiagonal algebra which was introduced by $\mathbf{F}$. Gilfeather and $D$. Larson. In this paper it is proved that if $\varphi: \operatorname{Alg} \mathscr{L} \rightarrow \operatorname{Alg} \mathscr{L}$ is a linear surjective isometry, then there exist unitary operators $W$ and $V$ such that $\varphi(A)=W A V$ for all $A \in$ $\mathrm{Alg} \mathscr{L}$.


Introduction. The study of reflexive, but not necessarily self-adjoint, algebras of Hilbert space operators has become one of the fastestgrowing specialties in operator theory. In this paper we study the linear surjective isometries of a certain class of reflexive algebras, which were introduced by F. Gilfeather, A. Hopenwasser and D. Larson [5]. These algebras have been found to be useful counterexamples to a number of plausible conjectures. In particular, these algebras have non-trivial cohomology [5], and they admit automorphisms which are not spatially implemented [2].

First we introduce the notation which is used in this paper. Let $\left\{e_{1}, e_{2}, \ldots, e_{2 n}\right\}$ and $\left\{e_{1}, e_{2}, \ldots\right\}$ be fixed bases of $2 n$-dimensional complex Hilbert space and separable infinite dimensional Hilbert space, respectively. If $x_{1}, x_{2}, \ldots, x_{k}$ are vectors in some Hilbert space, we denote by $\left[x_{1}, x_{2}, \ldots, x_{k}\right]$ the closed subspace spanned by the vectors $x_{1}, x_{2}, \ldots, x_{k}$.

Let $x$ and $y$ be two vectors in some Hilbert space. Then $(x, y)$ means the inner product of the vectors $x$ and $y$.

Let $H_{2 n}$ be $2 n$-dimensional Hilbert space. We denote by $\mathscr{L}_{2 n}$ the subspace lattice generated by the subspaces $\left[e_{1}\right],\left[e_{3}\right],\left[e_{5}\right], \ldots,\left[e_{2 n-1}\right]$, $\left[e_{1}, e_{2}, e_{3}\right],\left[e_{3}, e_{4}, e_{5}\right], \ldots,\left[e_{2 n-3}, e_{2 n-2}, e_{2 n-1}\right],\left[e_{1}, e_{2 n-1}, e_{2 n}\right]$.

By $\operatorname{Alg} \mathscr{L}_{2 n}=\Phi_{2 n}$ we mean the algebra of bounded operators which leave invariant all of the subspaces in $\mathscr{L}_{2 n}$. It is easy to see that all

