ISOMETRIES OF TRIDIAGONAL ALGEBRAS

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Let $\operatorname{Alg} \mathscr{L}$ be a tridiagonal algebra which was introduced by F. Gilfeather and D. Larson. In this paper it is proved that if $\varphi \colon \operatorname{Alg} \mathscr{L} \to \operatorname{Alg} \mathscr{L}$ is a linear surjective isometry, then there exist unitary operators W and V such that $\varphi(A) = WAV$ for all $A \in \operatorname{Alg} \mathscr{L}$.

Introduction. The study of reflexive, but not necessarily self-adjoint, algebras of Hilbert space operators has become one of the fastest-growing specialties in operator theory. In this paper we study the linear surjective isometries of a certain class of reflexive algebras, which were introduced by F. Gilfeather, A. Hopenwasser and D. Larson [5]. These algebras have been found to be useful counterexamples to a number of plausible conjectures. In particular, these algebras have non-trivial cohomology [5], and they admit automorphisms which are not spatially implemented [2].

First we introduce the notation which is used in this paper. Let $\{e_1, e_2, \ldots, e_{2n}\}$ and $\{e_1, e_2, \ldots\}$ be fixed bases of 2n-dimensional complex Hilbert space and separable infinite dimensional Hilbert space, respectively. If x_1, x_2, \ldots, x_k are vectors in some Hilbert space, we denote by $[x_1, x_2, \ldots, x_k]$ the closed subspace spanned by the vectors x_1, x_2, \ldots, x_k .

Let x and y be two vectors in some Hilbert space. Then (x, y) means the inner product of the vectors x and y.

Let H_{2n} be 2n-dimensional Hilbert space. We denote by \mathcal{L}_{2n} the subspace lattice generated by the subspaces $[e_1], [e_3], [e_5], \ldots, [e_{2n-1}], [e_1, e_2, e_3], [e_3, e_4, e_5], \ldots, [e_{2n-3}, e_{2n-2}, e_{2n-1}], [e_1, e_{2n-1}, e_{2n}].$

By Alg $\mathcal{L}_{2n} = \Phi_{2n}$ we mean the algebra of bounded operators which leave invariant all of the subspaces in \mathcal{L}_{2n} . It is easy to see that all