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APÉRY BASIS AND POLAR INVARIANTS OF PLANE CURVE SINGULARITIES

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Let C be an irreducible plane algebroid curve singularity over an algebraically closed field K, defined by a power series $f \in K[[X, Y]]$. In this paper, we study those power series $h \in K[[X, Y]]$ for which the intersection multiplicity $(f \cdot h) = \dim_K(K[[X, Y]]/(f, y))$ is an element of the Apéry basis of the value semigroup for C. We prove a factorization theorem for these power series, obtaining strong properties of their irreducible factors. In particular we show that some results by M. Merle and R. Ephraim are a special case of this theorem.

Introduction. In this paper we denote by K an algebraically closed field of arbitrary characteristic.

Let C be an irreducible plane algebroid curve over K (i.e. $C = \operatorname{Spec}(R)$, where $R = K[[X, Y]]/(f)$, with f irreducible). We will suppose $f \notin YK[[X, Y]]$ and we will write $n = \operatorname{Ord}_x(f(X, 0))$.

We will denote by $S(C)$ the semigroup of values of C (see [**2**], 11.0.1 and [**3**], 4.3.1), by $A_n = \{0 = a_0 < a_1 < \cdots < a_{n-1}\} = \{\min(S(C)n(k + n\mathbf{Z}_+); 0 \leq k \leq n-1\}$ the Apéry basis of $S(C)$ relative to n (see [**2**], 1.1.1) and by $\{v_0, \ldots, v_r\}$ the n-sequence in $S(C)$, where $v_0 = n$, and $v_i = \min\{v \in S(C); \gcd(v_0, v_1, \ldots, v_{i-1}) > \gcd(v_0, v_1, \ldots, v_{i-1}, v)\}$, $1 \leq i \leq r$ (see [**1**], 6.6, [**2**], 1.3.2 and [**6**]). (Note that $\gcd(v_0, \ldots, v_r) = 1$.)

The main objective of this work is the proof of the following theorem.

FACTORIZATION THEOREM. *Let $h \in K[[X, Y]]$ be such that $0 \leq k = \operatorname{Ord}_x(h(X, 0)) \leq n-1$. Then $(f \cdot h) \leq a_k$. Suppose $(f \cdot h) = a_k$. If $k = \sum_{0 \leq q \leq r} s_q(n/d_{q-1})$, where $d_q = \gcd(v_0, \ldots, v_q)$, $(d_0 = v_0 = n, d_r = 1)$, $0 \leq s_q \leq r$ and $0 \leq s_q \leq d_{q-1}/d_q$, then*

$$h = \prod_{1 \leq i \leq r} h_i \quad \textit{and} \quad h_i = \prod_{1 \leq j \leq m_i} h_{ij},$$

with h_{ij} either irreducible or unit in $K[[X, Y]]$, $1 \leq j \leq m_i$, $1 \leq i \leq r$, and

(1) $\sum_{1 \leq j \leq m_i} \operatorname{Ord}_x(h_j(X, 0)) = s_i(n/d_{i-1})$, $1 \leq i \leq r$.