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GENERALIZED RATIONAL CONVEXITY IN BANACH ALGEBRAS

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Let A be a complex commutative Banach algebra with identity and let F be a closed subset of its spectrum $X(A)$. There are several hulls, associated with F, which are useful in the study of approximation, interpolation and separation problems: the polynomially and rationally convex hulls are the most popular, the A-convex hull has also been considered and there are also holomorphically convex hulls. In this paper we introduce a family of hulls, denoted by $R_n(F)$, and we study some relations between these hulls and several known objects and invariants in commutative Banach algebras.

We study the relations between $R_n(F)$ and the generalized Shilov boundaries introduced by Basener [1] and Sibony [17], the topological stable rank, introduced by Rieffel [15], the dimension of $X(A)$, the minimal number of generators of A, etc. We describe briefly the contents of the paper. In §1 we introduce the hulls $R_n(F)$ and prove several basic properties of them. In §2 we consider those F's such that $R_n(F) = X(A)$; the intersection of all those F's, denoted by $\Gamma_n(A)$, seems to play a role similar to that of the generalized boundaries $S_n(A)$ of Basener and Sibony; we prove that $\Gamma_0(A) = S_0(A) \subset \Gamma_1(A) \subset S_1(A) \cdots$ but, in general, $\Gamma_n(A)$ is strictly contained in $S_n(A)$. In §3 we introduce the invariant $r(A) = \min\{n \geq 0 : R_n(F) = F$ for every $F\}$, which we call the *rationality* of A, and we relate it with the sets $\Gamma_n(A)$ and $S_n(A)$. In §4 we study the relationship between the topological stable rank of A with the new notions. In §5 we prove that $d(A) \leq r(A) + \gamma(A) \leq 2\gamma(A)$ where $d(A)$ is the (covering) dimension of $X(A)$ and $\gamma(A)$ is the minimal number of generators of A; the proof of this result uses known facts on Čech cohomology groups of some compact subsets of $\mathbb{C}^n$, due to Andreotti and Narasimhan and Duchamp and Stout (see [7]).

Section 6 contains a generalization of a result of Forelli [11] and §7 contains some results about the hulls R_n in the particular case of the algebra H^∞. Finally, in §8 we collect several open problems. There are several direct predecessors of this paper: one is a work of Csordas and Reiter [5], who introduced the hulls R_1 (which they call *L-sets*) and the