PACIFIC JOURNAL OF MATHEMATICS
Vol. 140, No. 1, 1989

COHOMOLOGY OPERATIONS FROM HIGHER PRODUCTS IN THE DERHAM COMPLEX

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We give a new construction of all Steenrood cyclic reduced powers $\mathscr{P}^i$ and of Pontrjagin-Thomas pth powers $\mathscr{B}_p$ for each prime p. The cohomology operations are indued by operations, analogous to the p-fold cup-i products, defined in the deRham complex of Cartan-Miller. These operations form a basis of all the cohomology operations derived from cyclic groups. This extends the construction of the Steenrod squares based on the analogue of the cup-i product in the deRham complex. From the construction of these new operations in the deRham complex it follows that the commutative cochain problem does not have a solution over the integers.

A key feature of cohomology is the existence of a commutative multiplication. This multiplication is induced by the cup product on cochains. While the product gives the cohomology a structure of a commutative ring, the cup product on the cochains is not commutative for arbitrary coefficients. The large number of cochains on any given space together with non-commutativity of the cup product makes any effective computation with cochains difficult.

Motivated by the rational deRham complex and its success in the rational homotopy theory, attempts were made to construct, for any space, a cochain complex with a commutative multiplication whose cohomology would be the cohomology of the space for any coefficient ring. One such construction was given by Cartan and Miller ([**1**], [**4**]). The commutative product in their complex induces the multiplication on cohomology with integer coefficients up to an additional factor, depending on the representing cochains. Another commutative cochain complex was constructed by Cenkl and Porter ("DeRham theorem with cubical forms," Pacific J. Math. **112** (1984), 35–48). In that paper we used a ring system for coefficients to solve the commutative cochain problem (as formulated in §5) for any space in terms of polynomial differential forms on simplices. The solution is the best possible in the sense of Theorem 2. The multiplication induced on cohomology by the commutative product of forms in that complex is exactly the usual one. A construction of the cohomology operations