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FANO BUNDLES OVER P^3 AND Q_3

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A vector bundle $\mathscr{E}$ is called Fano if its projectivization $P(\mathscr{E})$ is a Fano manifold. In this article we prove that Fano bundles exist only on Fano manifolds and discuss rank-2 Fano bundles over the projective space P^3 and a 3-dimensional smooth quadric Q_3.

Fano bundles appear naturally as we strive to construct examples of Fano manifolds of dimension ≥ 3; they form interesting yet accessible class of Fano n-folds. For example: among 87 types of Fano 3-folds with $b_2 \geq 2$ listed in **[13]** 22 types are ruled (i.e. obtained by projectivization of Fano bundles). Moreover some of the non-ruled manifolds listed there can be easily expressed as either finite covers of ruled 3-folds or divisors (or, more generally, complete intersections) in ruled Fano manifolds of higher dimension.

Let us mention another aspect of dealing with Fano bundles: it is how to determine whether or not a vector bundle is ample. This very fine property of a vector bundle cannot be determined by its numerical invariants, see **[7]**. Assuming the bundle to be stable helps to establish a sufficient condition for ampleness: **[10]**, **[17]**, which however is far from being necessary. In the present paper we take advantage of some already known facts about stable bundles with small Chern classes and determine that a bundle $\mathscr{E}$ is not ample by finding its jumping lines or sections of $\mathscr{E}(-k)$.

Let us note that some results of this paper have already been published, see remarks after the proofs of Theorems (1.6) and (2.1).

1. Fano bundles; preliminaries. Let $\mathscr{E}$ be a vector bundle of rank $r \geq 2$ on a smooth complex projective variety M. Let us recall that the tautological line bundle $\xi = \xi_{\mathscr{E}}$ on $V = P(\mathscr{E})$ is uniquely determined by the conditions $\xi_{\mathscr{E}}|F \approx \mathscr{O}_F(1)$ and $p_*\xi_{\mathscr{E}} = \mathscr{E}$. By p we have denoted the projection morphism of $V = P(\mathscr{E})$ onto M and by F—the fibre of p. Obviously, $F \cong P^{r-1}$ and $p\colon V \to M$ is a P^{r-1}-bundle. The Picard group of V can be expressed as a direct sum: $\operatorname{Pic} V \cong Z \cdot \xi_{\mathscr{E}} \oplus p^*(\operatorname{Pic} M)$. Replacing $\mathscr{E}$ by its twist with a line bundle $\mathscr{L}$ on M does not affect