

DEGREES AND FORMAL DEGREES
FOR DIVISION ALGEBRAS AND GL_n
OVER A p -ADIC FIELD

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We compute in the tame case, the degrees of the irreducible representations of a division algebra and the formal degrees of the discrete series of $GL(n)$ over a p -adic field and compare them.

1. Introduction. Let F be a p -adic field of characteristic zero, and let $G = GL_n(F)$. Throughout this paper, we assume that $(n, p) = 1$ (the tame case). The discrete series of G consists of (equivalence classes of) irreducible, unitary representations of G whose matrix coefficients are square integrable (mod Z), where Z is the center of G . The discrete series splits into two distinct classes ([HC2], [J]):

- (1) Supercuspidal representations: irreducible unitary representations whose matrix coefficients are compactly supported (mod Z);
- (2) Generalized special representations: irreducible unitary representations whose matrix coefficients are square integrable (mod Z), and which are subrepresentations of representations induced from a proper parabolic subgroup of G .

The supercuspidal representations of G were constructed by Howe [H2]. The first proof of the fact that all supercuspidal representations of G are contained in Howe's construction was given by Moy [M]. The generalized special representations of G were characterized by Bernstein-Zelevinsky ([BZ], [Z]). We note that the Bernstein-Zelevinsky construction uses the supercuspidal representations of $GL_m(F)$ where $m|n$ ($m < n$). Since $(m, p) = 1$ in the present case, the requisite supercuspidal representations can be obtained from Howe's construction.

The key to the study of the supercuspidal representations of G is the notion, due to Howe [H2], of an admissible character of an extension of degree n over F . In fact, the supercuspidal representations of G are parametrized by (conjugacy classes of) admissible characters of extensions of degree n over F , and generalized special representations are parametrized by (conjugacy classes of) admissible characters of