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PROPERTIES OF ERGODIC FLOWS ASSOCIATED TO PRODUCT ODOMETERS

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This paper deals with a property of group actions called approximate transitivity, defined by A. Connes and E. J. Woods. Their definition arose from solving the following interesting problem about von Neumann algebras: when is a von Neumann factor ITPFI? Here we study the property strictly from the ergodic theory point of view, and restate and prove part of their theorem in this context. Roughly speaking, a group action is approximately transitive if any finite collection of probability measures equivalent to the given measure on the space can be approximated by the convex hull of a single (equivalent) probability measure pushed around the space of measures under the action of the group. The main question one might ask about approximate transitivity is whether it is a new characterization of an already known property in ergodic theory, and if not, what are its properties? For example, Connes and Woods have already shown that approximate transitivity of a measure-preserving transformation implies zero entropy.

One way to understand the property is to reinterpret the theorem of Connes and Woods in the context of ergodic transformations. In this paper we prove that an odometer of product type has an approximately transitive Poincaré flow. In the first section of the paper we give the definitions, state a few known properties and prove some new properties of approximate transitivity. For example we prove that any AT flow is the factor action of a group action with simple spectrum. In the second section we give a short ergodic theoretic proof of one direction of the theorem of Connes and Woods.

That the converse to our result holds is part of the theorem of Connes and Woods [**CW**]; the proof is quite difficult and very long. A simpler ergodic theoretic proof would be desirable. By applying the full strength of the theorem of [**CW**] plus the links between orbit equivalence of ergodic transformations and isomorphism of hyperfinite von Neumann factors established by Krieger [**Kr**], the main result in this paper characterizes approximately transitive flows as those which are Poincaré flows of odometers with product measures.

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