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SOMMES EXPONENTIELLES DONT LA GEOMETRIE EST TRES BELLE: p-ADIC ESTIMATES

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In the present work we examine a family of multivariable exponential sums on a connected variety defined over a finite field.

0. Introduction. Let $K = \mathbb{F}_q$ be the field with q elements ($\operatorname{char} K = p \neq 2$, $q = p^{\ell}$), $\overline{x} \in K^{\times}$, $g_1, \dots, g_n$ positive integers relatively prime and prime to p ($n \geq 2$) and let $\mathscr{V}_{\overline{x}}$ be the variety defined over K by $\prod_{i=1}^{n} t_i^{g_i} = \overline{x}$. Let Ω be a complete algebraically closed field containing $\mathbb{Q}_p$, $\Theta: K \to \Omega^{\times}$ an additive character and for each $i \in \{1, \dots, n\}$ let $\chi_i: K^{\times} \to \Omega^{\times}$ be a multiplicative character. Let $\overline{c}_1, \dots, \overline{c}_n$ be non-zero elements of K, and let $\overline{f}(t) = \sum_{i=1}^{n} \overline{c}_i t_i^{k_i}$, where $k_1, \dots, k_n$ are positive integers prime to p. For each $m \in \mathbb{Z}_+$ let K_m be the extension of K of degree m. We consider the twisted exponential sums

$$(0.1) \quad S_m(\overline{f}, \mathscr{V}_{\overline{x}}) = \sum_{(\overline{t}_1, \dots, \overline{t}_n) \in \mathscr{V}_{\overline{x}}(K_n)} \prod_{i=1}^{n} \chi_i \circ N_{K_{m/K}}(\overline{t}_i) \times \Theta \circ \operatorname{Tr}_{K_{m/K}}(\overline{f}(\overline{t}))$$

and the associated L function:

$$(0.2) \qquad L = L(\overline{f}, \mathscr{V}_{\overline{x}}, T) = \exp\left(- \sum_{m=1}^{\infty} S_m(\overline{f}, \mathscr{V}_{\overline{x}}) T^m / m \right).$$

Our main results are the following:

A. We show that $L^{(-1)^n}$ is a polynomial of degree
$$h = \left(\sum_{i=1}^{n} g_i / k_i \right) \prod_{i=1}^{n} k_i.$$

B. We compute explicitly a lower bound for the Newton polygon of $L^{(-1)^n}$; this lower bound is independent of the prime number p and its endpoints coincide with those of the Newton polygon (Theorem 5.1 and Corollary 5.1).

C. Provided p lies in certain congruence classes, we show that our lower bound is in fact the exact Newton polygon of $L^{(-1)^n}$ (Theorem 5.3).