PACIFIC JOURNAL OF MATHEMATICS
Vol. 142, No. 1, 1990

UNITARY BORDISM OF CLASSIFYING SPACES OF QUATERNION GROUPS

ABDESLAM MESNAOUI

Let Γ_k be the generalized quaternion group of order 2^k. In this article we determine a set of generators for the $U_*(pt)$-module $\widetilde{U}_*(B\Gamma_k)$ and give all linear relations between them. Moreover their orders are calculated.

0. Introduction. In this article we first study the case $\Gamma_k = \Gamma$ the quaternion group of order 8. We recall that

$$\Gamma = \{\pm 1, \pm i, \pm j, \pm k\}, \qquad i^2 = j^2 = k^2 = -1,\ ij = k,\ jk = i,\ ki = ij.$$

Γ acts on S^{4n-3} by using $(n+1)\eta$ where η denotes the following unitary irreducible representation of Γ: $i \to \left(\begin{smallmatrix} i & 0 \\ 0 & -i \end{smallmatrix}\right)$, $j \to \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ and we get the element $w_{4n+3} = [S^{4n+3}/\Gamma, q] \in \widetilde{U}_{4n+3}(B\Gamma)$, q being the natural embedding: $S^{4n+3}/\Gamma \subset B\Gamma$. In **[6]** we have defined three elements of $\widetilde{U}^2(B\Gamma)$ denoted by A, B, C as Euler classes for MU of irreducible representations of Γ of dimension 1 over $\mathbb{C}$. Let $u_{4n+1} \in \widetilde{U}_{4n+1}(B\Gamma)$, $v_{4n+1} \in \widetilde{U}_{4n+1}(B\Gamma)$ be respectively $A \cap w_{4n+3}$ and $B \cap w_{4n+3}$. Our first result is:

THEOREM 2.2. *The set $\{u_{4n+1}, v_{4n+1}, w_{4n+3}\}_{n\geq 0}$ is a system of generators for the $U_*(pt)$-module $\widetilde{U}_*(B\Gamma)$.*
Their orders are given by:

THEOREM 2.6. *We have*: $\operatorname{ord} w_{4n+3} = 2^{2n+3}$.

THEOREM 2.8. *We have*: $\operatorname{ord} u_{4n+1} = \operatorname{ord} v_{4n+1} = 2^{n+1}$.

Now let Ω_* be $U^*(pt)[[Z]]$ graded by taking $\dim Z = 4$. If $P(Z) = \sum_{i\geq r} \alpha_i Z^i \in \Omega_n$ and $\alpha_r \neq 0$ then we denote $\nu(P) = 4r$. Let W, V_1, V_2 be the submodules of $\widetilde{U}_*(B\Gamma)$ generated respectively by $\{w_{4n+3}\}_{n\geq 0}$, $\{u_{4n+1}\}_{n\geq 0}$, $\{v_{4n+1}\}_{n\geq 0}$. The following result gives the $U_*(pt)$-module structure of $\widetilde{U}_*(B\Gamma)$ and uses the elements $T(Z) \in \Omega_4$, $J(Z) \in \Omega_0$ as defined in **[6]**, Section II.