PACIFIC JOURNAL OF MATHEMATICS
Vol. 142, No. 1, 1990

ASYMPTOTICS FOR CERTAIN WIENER INTEGRALS ASSOCIATED WITH HIGHER ORDER DIFFERENTIAL OPERATORS

RICHARD J. GRIEGO AND ANDRZEJ KORZENIOWSKI

The aim of this paper is to derive a large deviation principle for a certain class of higher order operators by combining the ideas of Donsker and Varadhan with the random evolution point of view of Griego and Hersh.

1. Introduction. It has been known for some time how to recover the principal eigenvalue for operators that generate Markov semigroups by means of the Large Deviation Principle of Donsker and Varadhan ([3], [4], [5]). The principal eigenvalues for such operators will be obtained as limits of certain functionals of Brownian motion.

We shall consider operators of the form $L = \frac{1}{2}\Delta_x + c(x)A_y$, where the Laplacian Δ_x is stochastic in the sense that it generates the Brownian motion semigroup; whereas A_y is analytic and does not correspond in general to a Markov process. The operator L can be interpreted as either the averaged result of randomization of the evolutions $c(x)A_y$ driven through the variable x in the coefficient $c(x)$ by Brownian motion or as a perturbation of the Laplacian by an operator-valued potential $V(x) = c(x)A_y$.

We follow the notation of [7], and we will recall some necessary facts. Let $A_y = \sum_{|\alpha|\leq 2r} a_\alpha(y)D^\alpha$ be a formally self-adjoint elliptic operator of order $2r$ on a bounded open set $G \subset R^m$, with domain $D(A_y)$ being a subset of the Sobolev space $H_{2r}(G)$, such that $(A_y g, g) \leq 0$ for $g \in D(A_y)$ with the inner product of $L^2(G)$.

In what follows we shall consider the following initial-boundary value problem

$$\text{(I.1)} \qquad \begin{aligned} &u_t(y, y', t) = A_y u, \qquad t > 0, y, y' \in G, \\ &u(y, y', 0) = \delta(y - y') \end{aligned}$$

subject to given homogeneous conditions on the boundary ∂G. We shall assume that the (fundamental) solution to this problem can be