

## HYPERHOLOMORPHIC FUNCTIONS AND HIGHER ORDER PARTIAL DIFFERENTIAL EQUATIONS IN THE PLANE

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We derive a Taylor formula for matrix-valued functions, in particular for hyperholomorphic functions. The latter functions are matrix-valued functions that satisfy a certain type of first order systems, for which we make no ellipticity assumption. For solutions of higher order linear partial differential equations with constant coefficients in the plane we show the existence of hyperconjugates, an obvious generalization of harmonic conjugates in complex analysis. By way of hyperconjugates we find series expansions for solutions of partial differential equations in terms of polynomial solutions. These polynomials form a basis for real analytic solutions at the origin. An algorithm for obtaining all such polynomials is summarized at the end. This paper continues in the tradition of hypercomplex analysis.

**1. Matrix-valued functions.** Matrix-valued functions are freely added or multiplied whenever their sizes are compatible. In writing the product  $FG$  we automatically assume the number of columns of  $G$  to be equal to the number of rows of  $G$ . We shall not single out any particular class of matrix-valued functions to form an algebra. The underlying scalars for the matrices can be real, complex, or perhaps even elements of a Banach algebra. Most of the basic concepts in the calculus of scalar-valued functions can be readily extended to matrix-valued functions by means of “componentwise applications”. However, certain complications are expected because matrix multiplications are not commutative. Although we need not restrict to two independent variables, we will do so in order to simplify our presentation.

Let  $F$  belong to class  $C^1$  in some unspecified domain, namely every component function  $f_{ij}(x, y)$  has continuous first order partial derivatives, then the *differential* of  $F$  or  $(df_{ij})$  is conveniently expressed as  $dF = F_x dx + F_y dy$  where the subscripts represent componentwise partial differentiation. More generally we consider a *differential form*  $P(x, y) dx + Q(x, y) dy$ , which is said to be *exact* if there exists an  $F$  such that  $dF = P dx + Q dy$ , or equivalently,  $P = F_x$  and  $Q = F_y$ .