PACIFIC JOURNAL OF MATHEMATICS
Vol. 142, No. 2, 1990

BLASCHKE COCYCLES AND GENERATORS

JUN-ICHI TANAKA

Dedicated to Professor Shōzo Koshi on his 60th birthday

Using a local product decomposition, we establish a certain class of Blaschke cocycles with the property that a simply invariant subspace has a single generator if and only if its cocycle is cohomologous to one of this class. Some applications are also obtained. We show, among other things, every simply invariant subspace is approximated by a singly generated one as near as desired.

1. Preliminaries. Let Γ be a dense subgroup of the real line R, endowed with the discrete topology, and let K be the dual group of Γ. For each t in R, e_t denotes the element of K defined by $e_t(\lambda) = e^{i\lambda t}$ for any λ in Γ. Then the mapping from t to e_t embeds R continuously onto a dense subgroup of K. Choose and fix a positive γ in Γ, and let K_γ be the subgroup consisting of all x in K such that $x(\gamma) = 1$. Then K may be identified measure theoretically, and almost topologically, with $K_\gamma \times [0, 2\pi/\gamma)$ via the mapping $y + e_s$ to (y, s). We assume, for simplicity, that 2π lies in Γ throughout the paper. Thus K may be regarded as $K_{2\pi} \times [0, 1)$. This local product decomposition is very useful for understanding the group K. We denote by σ and σ_1 the normalized Haar measures on K and $K_{2\pi}$, respectively. Then $d\sigma$ is carried by the above mapping to the restriction of $d\sigma_1 \times dt$ to $K_{2\pi} \times [0, 1)$.

A Borel function V on $K_{2\pi} \times R$ is *automorphic* if $V(y, t+1) = V(y + e_1, t)$ for $d\sigma_1 \times dt$-a.e. (y, t) in $K_{2\pi} \times R$. Every Borel function φ on K has the *automorphic extension* $\varphi^\#$ to $K_{2\pi} \times R$ by

$$\varphi^\#(y, t) = \varphi(y + e_{[\![t]\!]}, t - [\![t]\!])$$

for each (y, t) in $K_{2\pi} \times R$, where $[\![t]\!]$ denotes the largest integer not exceeding t. Conversely, if V is automorphic on $K_{2\pi} \times R$, then there is a function φ on K of which the automorphic extension is V, since V is determined by its values on $K_{2\pi} \times [0, 1)$.

A function φ in $L^1(\sigma)$ is *analytic* if its Fourier coefficients

$$a_\lambda(\varphi) = \int_K \overline{\chi}_\lambda(x)\varphi(x)\, d\sigma(x)$$