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NORMAL STRUCTURE IN BOCHNER L^p-SPACES

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It is shown that, for $1 < p < \infty$, the Bochner L^p-space $L^p(\mu, X)$ has normal structure exactly when X has normal structure. With this result, normal structure in Bochner L^p-spaces is completely characterized except in one seemingly simple setting.

The concept of normal structure, a geometric property of sets in normed linear spaces, was introduced in 1948 by M. S. Brodskiĭ and D. P. Mil′man in order to study the existence of common fixed points of certain sets of isometries. Since then, normal structure has been studied both as a purely geometric property of normed linear spaces and as a tool in fixed point theory [**1, 4, 5, 6**]. In 1968, L. P. Belluce, W. A. Kirk, and E. F. Steiner [**1**] proved that the l^∞-direct sum of two normed linear spaces with normal structure has normal structure. They were not however able to decide if normal structure is preserved under an l^p-direct sum of two normed linear spaces for $1 \leq p < \infty$. In 1984, T. Landes [**5**] proved that, if $1 < p < \infty$, normal structure is preserved under finite or infinite l^p-direct sums. In this article, the corresponding theorem is proven in the nondiscrete setting; consequently it is shown that, if (Ω, Σ, μ) is any measure space and $1 < p < \infty$, the Bochner L^p-space $L^p(\mu, X)$ has normal structure exactly when X has normal structure.

A normed linear space X has *normal structure* if, for each closed bounded convex set K in X that contains more than one point, there is a point p in K such that $\sup\{\|p-x\| : x \in K\}$ is less than the diameter of K; such a point p is called a *nondiametral point* in K. Brodskiĭ and Mil′man [**2**] proved that a space X fails to have normal structure if and only if there is a nonconstant bounded sequence (x_n) in X such that the distance from x_{n+1} to the convex hull of $\{x_1, \ldots, x_n\}$ tends to the diameter of the set $\{x_k : k \in N\}$; such a sequence is called a *diametral sequence* in X. One consequence of this fact is that normal structure is a separably-determined property. Several more conditions equivalent to normal structure are given in [**5**] and in Lemma 3 below.

The notation used in this paper is, with perhaps two exceptions, standard. The two exceptions are as follows: if (x_n) is a sequence in a