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BANACH ALGEBRAS ASSOCIATED WITH SPHERICAL REPRESENTATIONS OF THE FREE GROUP

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We prove that any spherical representation of the free group $\mathbb{F}$ weakly contains the regular representation. Moreover C_π^*, the C^*-algebra associated with the spherical representation π, is a compact extension of the reduced C^*-algebra of $\mathbb{F}$. We also show that the standard projection onto radial functions admits extensions to C_π^* for a class of representations π of $\mathbb{F}$ which includes spherical representations, as well as the regular representation and the universal representation.

Introduction. Let $\mathbb{F}_r$ be a free group on r generators $x_1, \dots, x_r$. Let μ_1 be the finitely supported probability measure equidistributed on $\{x_1^{\pm 1}, x_2^{\pm 1}, \dots, x_r^{\pm 1}\}$. The operator of convolution by μ_1 is the analogue of the Laplace-Beltrami operator on Riemann rank one symmetric spaces. By **[11]** the l^1-spectrum of μ_1 can be identified with the ellipse $E = \{z = x + iy\colon x^2 + (\frac{r}{r-1}y)^2 \leq 1)$. Any point z of E corresponds in one-to-one fashion to a spherical function ϕ_z the eigenfunction of μ_1 with eigenvalue z. We refer to **[11]**, [7] for this subject.

For real z spherical functions are positive definite and give rise to unitary representations of $\mathbb{F}_r$. Basing our argument on a particular realisation of these representations and on the simplicity of the reduced C^*-algebra of $\mathbb{F}_r$ **[10]**, we prove that all spherical representations weakly contain the regular representation. Moreover the C^*-algebras associated with spherical representations are compact extensions of $C^*_{\text{red}}(\mathbb{F}_r)$, the C^*-algebra associated with the regular representation.

Finally we consider the standard projection onto radial functions on $\mathbb{F}_r$ and we prove that it is bounded on any C^*-algebra associated with spherical functions.

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Spherical representations. Let $\mathbb{F}_r$ be a free group on r generators $x_1, x_2, \dots, x_r$, $r \geq 2$. Any element x of $\mathbb{F}_r$ may be uniquely ex-