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RESIDUE CLASS DOMAINS OF THE RING OF CONVERGENT SEQUENCES AND OF $C^\infty([0, 1], \mathbf{R})$

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We show that there are exactly 10 residue class domains of c, the ring of real convergent sequences. We also classify some of the residue class domains of $C^\infty([0, 1], \mathbf{R})$.

Introduction. The residue class domains of $C(X, \mathbf{R})$ (the ring of real valued continuous functions on a topological space X) have been extensively studied by Kohls **[16]**, Gillman and Jerison **[10]**, and others.

In **[6]**, Cherlin and Dickmann began the study of the residue class domains of $C(\mathbf{N}^*, \mathbf{R})$, where $\mathbf{N}^*$ is the one-point compactification of $\mathbf{N}$. Clearly $C(\mathbf{N}^*, \mathbf{R})$ is isomorphic to c, the ring of real convergent sequences. In **[8]**, Cherlin and Dickmann (and Louveau) showed that there exist non-maximal prime ideals p of $C(\mathbf{N}^*, \mathbf{R})$ such that $C(\mathbf{N}^*, \mathbf{R})/p$ is a real closed valuation ring. They asked about the other prime ideals. Both the author's dissertation (written under Cherlin's guidance) and this paper grew out of that question. We deal with two main problems:

(1) Classify all the residue class domains of $C(\mathbf{N}^*, \mathbf{R})$, assuming the continuum hypothesis.

(2) Classify the residue class domains of $C^\infty([0, 1], \mathbf{R})$.

(The reader may well ask, "What about $C([0, 1], \mathbf{R})$?" We touch this question very lightly. Cherlin and Dickmann, in §4 of **[8]**, go into it more deeply.)

We completely solve problem 1 in this paper. We show that there are exactly ten residue class domains of $C(\mathbf{N}^*, \mathbf{R})$.

We can classify these domains in the following manner, considering c rather than $C(\mathbf{N}^*, \mathbf{R})$:

First, one of these conditions will hold for $(l^\infty/p - c/p)$; it will either be empty, have a non-empty countable coinitial subset, or be non-empty with no countable coinitial subset.

Second, one of these conditions will hold on neighborhoods of zero in c/p; there will either be a countable coinitial subset of c/p of the form $\{[f]^k\}_{k=1}^\infty$, or a countable coinitial subset $\{[f_m]\}_{m=1}^\infty$ of c/p