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THE ISOMETRIES OF $H^\infty(E)$

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Let E be a uniformly convex and uniformly smooth complex Banach space. We prove that every onto isometry T on $H^\infty(E)$ is of the form

$$(TF)(z) = \mathscr{T}(F(t(z))) \qquad (F \in H^\infty(E),\ |z| < 1),$$

where $\mathscr{T}$ is an isometry from E onto E and t is a conformal map of the unit disc onto itself.

1. Introduction. Let H^∞ denote the set of all bounded analytic functions in the open unit disc with the norm $\|f\|_\infty = \sup_{|z|<1} |f(z)|$. Since H^∞ is a semi-simple commutative Banach algebra, the Gelfand transform $(f \to \hat{f})$ is an isometry from H^∞ onto a subalgebra $\widetilde{M}$ of $C(Y)$ where Y is the maximal ideal space of H^∞. One can show **[L-R-W]**:

To every extreme point L of the unit ball $(H^\infty)^*$ there corresponds a complex number α of absolute value 1 and a point $y \in Y$ (indeed, y is an element in the Choquet boundary of Y) such that

$$Lf = \alpha \hat{f}(y) \qquad (f \in H^\infty).$$

Using this result, K. deLeeuw, W. Rudin and J. Wermer (**[L-R-W]**; also see **[N]**) proved that every linear isometry T of H^∞ onto H^∞ is of the form

$$(Tf)(z) = \alpha f(t(z)) \qquad (f \in H^\infty,\ |z| < 1),$$

where α is a complex number of absolute value 1 and t is a conformal mapping of the unit disc onto itself. If E is a complex Banach space, then $H^\infty(E)$ denotes the set of all E-valued bounded analytic functions defined on the open unit disk Δ. We will show that there is a linear isometry from $H^\infty(E)$ onto a subspace $\widetilde{M}$ of $C((Y,\ \text{weak}^*\ \text{topology}) \times (U,\ \text{norm topology}))$ where U is the unit ball of E^*. M. Cambern **[C1]** proved that: If E is a finite dimensional complex Hilbert space, then

to every extreme point L of the unit ball of $(H^\infty)^*$ there corresponds a point y in the Choquet boundary $B \subset Y$ of H^∞ and a point e^* in