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TRIANGLE IDENTITIES AND SYMMETRIES OF A SUBSHIFT OF FINITE TYPE

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We prove the group $\mathrm{Aut}(\sigma_A)$ of symmetries of a subshift of finite type is isomorphic to the fundamental group of the space $\mathrm{RS}(\mathscr{E})$ of strong shift equivalences built from the algebraic RS Triangle Identities for zero-one matrices which arise from triangles in the contractable simplicial complex of Markov partitions. Moreover, we show the higher homotopy groups of $\mathrm{RS}(\mathscr{E})$ are zero. $\mathrm{RS}(\mathscr{E})$ is therefore homotopy equivalent to the classifying space of $\mathrm{Aut}(\sigma_A)$.

1. Introduction and statement of results. First we briefly review Williams' strong shift equivalence criterion for conjugacy of subshifts of finite type. See [**3**, **4**, **8**]. Let $A: \mathscr{S}\times\mathscr{S} \to \{0, 1\}$ and $B: \mathscr{T}\times\mathscr{T} \to \{0, 1\}$ be zero-one matrices on the finite state spaces $\mathscr{S}$ and $\mathscr{T}$. An *elementary strong shift equivalence*

$$(R, S): A \to B$$

is a pair of zero-one matrices $R: \mathscr{S}\times\mathscr{T} \to \{0, 1\}$ and $S: \mathscr{T}\times\mathscr{S} \to \{0, 1\}$ satisfying

$$RS = A \quad \text{and} \quad SR = B.$$

Let (X_A, σ_A) and (X_B, σ_B) be the subshifts of finite type (SFT) constructed from A and B respectively. The strong shift equivalence (R, S) gives rise to an *elementary symbolic conjugacy*

$$c(R, S): X_A \to X_B$$

defined as follows: Let $x = \{x_n\}$ be in X_A. Then $y = c(R, S)(x)$ is the unique point $y = \{y_n\}$ in X_B such that $1 = A(x_n, x_{n+1}) = R(x_n, y_n)S(y_n, x_{n+1})$ for all n. Similarly, one has

$$c(S, R): X_B \to X_A$$

and it is easy to verify the identities

$$c(S, R)c(R, S) = \sigma_A \quad \text{and} \quad c(R, S)c(S, R) = \sigma_B$$

which show that $c(R, S)$ and $c(S, R)$ are conjugacies. More generally, let $\mathscr{E}$ denote the set of zero-one matrices on finite state spaces. We shall assume that any matrix in $\mathscr{E}$ has at least one non-zero entry