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VECTOR SINGULAR INTEGRAL OPERATORS ON A LOCAL FIELD

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A theory of vector singular integral operators in the context of the local fields, is established. Applications to maximal functions, a diagonal multiplier theorem of Mihlin-Hörmander type and applications to Besov and Hardy-Sobolev spaces are given.

Introduction. The theory of the vector singular operators with operator valued kernels on Euclidean space was treated systematically by Rubio de Francia, Ruiz and Torrea [**6**] (see also Garcia-Cuerva and Rubio de Francia [**3**]). On the other hand, the classical singular integral operators of the Calderón-Zygmund type on finite product of local fields were considered by Phillips and Taibleson [**5**].

The goal of the present paper is to give a version for local fields of some results of Francia-Ruiz-Torrea [**6**] that generalize from several perspectives the quoted paper by Phillips-Taibleson.

The contents of the paper is as follows. We begin in §1 some basic notations, definitions and results that we can find in [**9**]. In §2 we state an inequality of Fefferman-Stein type and, we apply it to obtain an interpolation theorem of Marcinkiewicz-Riviere type. The main results are in §3 where we state the version of the integral singular operator theorem given in [**6**], for local fields, giving also sequential extensions. Next in §4 we obtain maximal inequalities of F. Zó and Fefferman-Stein type. A diagonal multiplier theorem of Mihlin-Hörmander type (for the Euclidean case see Triebel [**11**]) that generalize the scalar multiplier theorem of Taibleson [**8**] is given in §5. Finally, in §6 we give applications of some results obtained in the foregoing sections to Besov and Hardy-Sobolev spaces in local fields.

The extension of all results in this paper for a finite product of local fields will be an immediate consequence of a M. H. Taibleson's theorem (see [**10**], pp. 548–549) which states that, if $\mathbb{K}$ is a local field and d is an integer greater than 1, then $\mathbb{K}^d e$, the d-dimensional vector space over $\mathbb{K}$, has a field structure, as a local field, which is compatible with the usual vector space norm of $\mathbb{K}^d$.

1. Preliminaries. A local field is any locally compact, non-discrete and totally disconnected field. Let $\mathbb{K}$ be a fixed local field and dx a