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# REDUCTIONS OF FILTRATIONS

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**Let $\phi = \{\phi(n)\}_{n \geq 0}$ be a filtration on a ring $R$. Then the concept of a reduction of $\phi$ is introduced, several basic properties of such reductions are established, and then these results are used to characterize analytically unramified semi-local rings and locally quasi-unmixed Noetherian rings.**

**1. Introduction.** Reductions of ideals were introduced in **[6]** and they have proved to be very useful in many research problems. Recently, reductions of modules were introduced and developed in **[16]**, and their "dual" concept was investigated in **[20]**. Also, there have been several recent papers in which a number of important theorems for ideals in Noetherian rings have been extended to Noetherian filtrations (for example, see **[1, 8, 9, 14, 15, 18]**). (Filtrations are generalizations of the sequence of powers of a given ideal, and there are many important filtrations (such as the sequence $\{q^{(n)}\}_{n \geq 0}$ of symbolic powers of a primary ideal $q$ and the sequence $\{(I^n)_a\}_{n \geq 0}$ of integral closures of the powers of an ideal $I$) which are generally not powers of an ideal, but which are quite often Noetherian filtrations. So extension of these results to filtrations is of some interest and importance.)

In §2 we introduce reductions of filtrations and show that many of the basic properties of reductions of ideals have a natural extension to reductions of filtrations. (Actually, basic reductions of Noetherian filtrations were first considered and briefly used in **[15]**, but general reductions and their properties were not considered in **[15]**.) Among these properties is the very useful result that if $\phi$ and $\gamma$ are filtrations on a Noetherian ring $R$, then $\phi$ is a reduction of $\gamma$ if and only if the Rees ring of $R$ with respect to $\gamma$ is a finite integral extension ring of the Rees ring of $R$ with respect to $\phi$. It readily follows from this that if $\phi$ is a reduction of $\gamma$, then $\phi$ is Noetherian if and only if $\gamma$ is Noetherian. Also, if $\phi$ is Noetherian, then $\phi$ is a reduction of $\gamma$ if and only if $\phi \leq \gamma$ and $\phi$ and $\gamma$ determine linearly equivalent ideal topologies on $R$ if and only if $\gamma$ is a Noetherian filtration between $\phi$ and $\phi_w$, the weak integral closure of $\phi$, and then there exists a