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ON THE DISTRIBUTION OF WEIERSTRASS POINTS ON IRREDUCIBLE RATIONAL NODAL CURVES

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Let X be an irreducible rational nodal curve of arithmetic genus $g \geq 2$, and let $\mathscr{L}$ be a non-special, effective invertible sheaf on X. Let $W(\mathscr{L})$ denote the set of smooth Weierstrass points of $\mathscr{L}$ and all its positive tensor powers on X. In this paper, we study the distribution of $W(\mathscr{L})$ on X. In particular, we will show that $W(\mathscr{L})$ is not dense on X, generalizing an example of R. F. Lax.

1. Introduction. In a recent series of papers ([**2**], [**3**], [**4**]), R. F. Lax and C. Widland have defined Weierstrass points for invertible sheaves on integral, projective Gorenstein curves over $\mathbf{C}$. They use a method generalizing the classical definition of the Weierstrass points of the canonical sheaf on a smooth curve via Wronskians. In particular, they show that if X is an integral, projective Gorenstein curve, and $\mathscr{L}$ is an invertible sheaf on X, then a smooth point $P \in X$ is a Weierstrass point of $\mathscr{L}$ if and only if

$$\dim H^0(X, \mathscr{L}(-sP)) > 0,$$

where $s = \dim H^0(X, \mathscr{L})$. On the other hand, if $s \geq 2$, the singular points of X are automatically Weierstrass points of $\mathscr{L}$ of high Weierstrass weight. (See Propositions 2 and 3 of [**3**].)

The goal of the present note is to prove a general result about the distribution of the smooth Weierstrass points of an invertible sheaf $\mathscr{L}$ and all its positive tensor powers in the case that X is an irreducible rational nodal curve. This particular question was suggested by an example in [**3**], in which it is shown that for a particular $\mathscr{L}$ on a particular rational nodal curve of arithmetic genus 2, the set

$$W(\mathscr{L}) = \{P \in X \mid X \text{ is a smooth Weierstrass point of } \mathscr{L}^{\otimes n} \text{ for some } n \geq 1\}$$

avoids a small disk in the normalization of X (that is, $\mathbb{P}^1$). This situation is quite different from the case of smooth curves X, where B. Olsen ([**6**]) had previously shown that if $\deg(\mathscr{L}) > 0$, then the analogous set $W(\mathscr{L})$ is dense in the complex topology on X.