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UNIQUENESS FOR A NONLINEAR ABSTRACT CAUCHY PROBLEM

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Let H be a complex Hilbert space, and let A be a linear, unbounded operator defined on a domain D in H. We show that the Cauchy problem for differential equations and inequalities involving the operator $d^n u/dt^n - Au$ as the principal part have at most one solution. No symmetry conditions are placed on the operator A.

1. Introduction. Let H be a complex Hilbert space and let A be a linear (in general, unbounded) operator defined on a domain D in H. We consider differential inequalities in which the principal part is given by

$$Lu = d^n u/dt^n - Au \tag{1.1}$$

where n is a fixed positive integer and neither symmetry nor semiboundedness conditions are placed on the operator A although there will be restrictions placed on the symmetric and antisymmetric parts of A. Our purpose, in short, is to extend the uniqueness results of Hile and Protter [**5**], where $n = 1, 2$ in (1.1) and A depends on t, to operators L in which n is arbitrary and A is independent of t. Furthermore, we obtain the uniqueness results of [**10**] as a special case. The method employed, developed originally in the study of elliptic equations (see e.g., [**12**]) and later extended to parabolic equations [**8**], is essentially the same as that used by Hile and Protter [**5**]. This same weighted L_2 argument has been employed in other similar contexts where A has been a specific partial differential operator. (See e.g., [**6**, **7**, **8**].)

Levine [**10**], generalizing previous results of Murray [**11**], proved that the only solution of $Lu = 0$ with $u(0) = u'(0) = \cdots = u^{(n-1)}(0) = 0$ is the zero function, provided the operator A is either symmetric or antisymmetric. The only other results for the operator L in which $n > 2$ and A is unbounded seem to be those of Fattorini [**2**, **3**] and Fattorini and Radnitz [**4**] who study the equation $Lu = 0$ under complete and incomplete Cauchy data. As Levine [**10**] points out, equations involving L in which A is bounded, or $n \leq 2$ and