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INFINITESIMAL RIGIDITY OF ALMOST-CONVEX ORIENTED POLYHEDRA OF ARBITRARY EULER CHARACTERISTIC

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This paper introduces a new method for proving the infinitesimal rigidity of a broad class of polyhedra, the caps-with-collars and their projective (Darboux) transforms, which include, as special cases, the traditional closed convex polyhedra of Cauchy and the refined closed convex and open convex polyhedra of Alexandrov with total curvature 2π. By definition, a cap-with-collars consists of an oriented generalized polyhedral cap with cylindrical polyhedral collars attached to the boundary. The spherical image of the cap (by unit normals coherent to the orientation) must lie in some hemisphere with the collars glued to the boundary of the cap so that their faces are parallel to the polar vector of the hemisphere. Moreover, caps are required to satisfy a local convexity condition, called edge-convexity, which is weaker than traditional convexity. An edge-convex polyhedron need not have a local supporting plane at each point. This allows great topological and morphological variety. A cap-with-collars can have arbitrary Euler characteristic. Among the examples given some are nonconvex; some are surfaces of genus greater than one; some are self-intersecting surfaces; some have branch points and some have pinch points.

1. Introduction. Consider the examples of polyhedral surfaces illustrated in Figure 1.

Figure 1A shows a nonconvex cap-with-collar with an edge-convex cap (shaded faces), and its closed Darboux transform. (The light lines are an Alexandrov refinement of the cap, dividing its plane faces.) Figure 1B shows the closed transform of a nonconvex cap-with-collar with a branchpoint at the central vertex of the cap and one at the conical vertex (i.e., the transform of the point at infinity on the collar). In Figure 1C the polyhedron is an immersion of a closed polyhedron of genus 2 (see Example 3, Section 5). Figure 1D is a pentagram bipyramid, and Figure 1E is a pinched sphere, where the curved lines indicate a convex polyhedral cap. The proofs of their infinitesimal rigidity follow from the basic theorem of the paper and its extensions (Theorem B$'$, Section 5 and Theorem C, Section 6):

THEOREM B. *Let C be a cap with Alexandrov refinement C', satisfying the hemisphere condition $\mathbf{c} \cdot \mathbf{n} > 0$ and having exactly one*