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THE ABSOLUTE INVARIANCE OF CONSERVATION LAWS

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The only known absolute invariant for a general system of pde is its maximal character. The purpose of this paper is to prove that conservation laws are also absolute invariants, that is, they are preserved under partial prolongations in a natural way. We also show that the property, closely related to soliton behavior, of having an infinite number of conservation laws is an absolute invariant.

I. Introduction and background. Partial prolongations of systems of pde were introduced by E. Cartan to define when two continuous infinite dimensional transformation pseudogroups are equivalent as abstract groups [**1**, p. 625ff]. With this relation he was able to classify the simple infinite pseudogroups [**1**, p. 857ff].

In [**1**, p. 1133] Cartan studied partial prolongations of general systems of pde and used them to make precise the notion due to D. Hilbert of when two systems possess a one-to-one correspondence between their solutions [**6**]. Two systems are "absolutely equivalent" when they can be joined by a finite sequence of systems where for each adjacent pair in the sequence, one is a partial prolongation of the other.

Thus, the main use of partial prolongations is geometric: a property associated with systems of pde is an "absolute invariant" property when it is shared by both systems in any partial prolongation.

In a series of papers the author showed that several classical concepts (hyperbolicity, characteristics) are not, in fact, absoluted invariants [**7**, **8**, **9**, **10**, **11**]. This means that these concepts are not necessarily intrinsically related to the systems they are defined on: if one changes the form of the system by a partial prolongation, such concepts may change or no longer exist. Any theory based on these non-absolute invariants may predict different kinds of behavior for systems which are in fact completely and naturally equivalent.

In physics the problem is even more crucial. Concepts which are not absolute invariants of the systems of pde that describe the physical