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APPROXIMATING EQUIVARIANT MAPPING SPACES

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Let SV and S^V be the unit sphere and one-point compactification of the unitary representation V of the finite group G. One has the associated self-mapping G-spaces $\mathcal{U}(SV, SV)$ and $\Omega^V S^V$ respectively, the first consisting of unbased maps and the second of based maps. It is the goal of this paper to describe homotopy approximations of these loop spaces (as examples of a more general class of G-spaces), along the lines of the group completion approximations of Segal, McDuff and Hauschild. We then apply these approximations to obtain splittings and Hopf space structures for several spaces.

Introduction. Let G be a finite group and let V be a finite dimensional unitary representation of G with unit sphere SV. Let $\mathcal{U}(SV, SV)$ be the G-space of unbased maps $SV \to SV$, where G acts by conjugation. If S^V is the one point compactification of V, denote the G-space of based maps $S^V \to S^V$ by $\Omega^V S^V$. It is the goal of this paper to describe properties of these loop spaces, viewed as examples of a more general class of G-spaces, through the use of G-homotopy approximations.

May [**M1**] first used configurations of "little cubes" to obtain a nonequivariant homotopy approximation of $\Omega^n\Sigma^n Z$ for a connected based CW complex Z. He also showed that the little cubes construction can be replaced by an analogous configuration space construction. Segal [**S**] then showed that $\Omega^n S^n$ $(= \Omega^n\Sigma^n S^0)$ is the group completion of the space of configurations of points in the unit disc D^n. Formally, a group completion is a map $\alpha: X \to Y$ from a Hopf space to a group-like Hopf space such that α coincides with localization at $\pi_0(X)$ in homology with field coefficients. May's use of configurations of little cubes rather than points had the effect of greatly simplifying the arguments, although the corresponding approximating spaces are homotopy equivalent. McDuff [**M2**] generalized Segal's result to obtain approximations up to group completion of spaces of sections of certain sphere bundles over a compact manifold M. Generalizing May's result, Caruso and Waner [**CW1**] showed that $\Omega^n\Sigma^n Z$ is homotopy equivalent to a space of configurations of "positive and negative little cubes" in the n-disc D^n, whether or not Z is connected. (McDuff