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ON SOME TOTALLY ERGODIC FUNCTIONS

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Dedicated to Dagmara Klim and Nina Tomaszewska

We study some classes of totally ergodic functions on locally compact Abelian groups. Among other things, we establish the following result: If R is a locally compact commutative ring, $\mathscr{R}$ is the additive group of R, χ is a continuous character of $\mathscr{R}$, and p is the function from $\mathscr{R}^n$ $(n \in \mathbb{N})$ into $\mathscr{R}$ induced by a polynomial of n variables with coefficients in R, then the function $\chi \circ p$ either is a trigonometric polynomial on $\mathscr{R}^n$ or all of its Fourier-Bohr coefficients with respect to any Banach mean on $L^\infty(\mathscr{R}^n)$ vanish.

1. Introduction. Let G be a locally compact Abelian group, λ_G be the Haar measure in G, and $L^\infty(G)$ be the space of all classes of complex-valued λ_G-measurable λ_G-essentially bounded functions on G endowed with the λ_G-essential supremum norm.

A linear continuous functional m on $L^\infty(G)$ is called a Banach mean on $L^\infty(G)$ if it satisfies the following conditions:

(i) $m(1) = 1 = \|m\|$,
(ii) $m(T_a f) = m(f)$ for each $a \in G$ and each $f \in L^\infty(G)$, where $T_a f(b) = f(a+b)$ for any $b \in G$.

When G is finite, there is precisely one Banach mean on $L^\infty(G)$. When G is infinite, then the set of all Banach means on $L^\infty(G)$ has at least the cardinality of the continuum (cf. [**6**, Propositions 22.26 and 22.41]).

Let $\widehat{G}$ be the dual group of G. Given $f \in L^\infty(G)$, $\chi \in \widehat{G}$, and a Banach mean m on $L^\infty(G)$, let $\mathscr{F}_m f(\chi)$ stand for the Fourier-Bohr coefficient of f at χ with respect to m, defined to be $m(f\bar{\chi})$.

A function f in $L^\infty(G)$ is said to be ergodic if its mean value $m(f)$ is independent of the choice of the Banach mean m on $L^\infty(G)$. A function f in $L^\infty(G)$ is said to be totally ergodic if, for every $\chi \in \widehat{G}$, the function $f\chi$ is ergodic (cf. [**7**, **8**]). Let $E(G)$ be the space of all ergodic functions in $L^\infty(G)$, $TE(G)$ be the space of all totally ergodic functions in $L^\infty(G)$, and $TE_0(G)$ be the subspace of $TE(G)$ consisting of those $f \in L^\infty(G)$ for which $\mathscr{F}_m f(\chi) = 0$ for any $\chi \in \widehat{G}$ and any Banach mean m on $L^\infty(G)$. Let $P(G)$ be the space of all