PACIFIC JOURNAL OF MATHEMATICS
Vol. 144, No. 2, 1990

AN ALGEBRAICALLY DERIVED q-ANALOGUE OF A CHARACTER SUM ASSOCIATED WITH A CLASS OF SEMIREGULAR PERMUTATIONS

G. E. ANDREWS AND D. M. JACKSON

The group algebra of the symmetric group can be used to determine the cycle structure of permutations which are obtained as products of designated conjugacy classes. Such matters arise, for example, in certain topological questions and in the embedding of graphs on orientable surfaces. We consider a set of permutations restricted by cycle structure, and use basic hypergeometric series to derive q-analogues associated with the generating functions for the numbers of such permutations. The expressions which are derived pose a number of combinatorial questions about their connexion with the Hecke algebra of the symmetric group.

1. Introduction and background to the problem. A permutation is said to be *p-semiregular* if all of its cycles have the same length p. In this paper we derive a q-analogue for the number $e(k, p)$ of p-semiregular permutations, with k cycles, which are the product of a designated full-cycle and a fixed point free involution. Such permutations occur in several areas of combinatorial theory ([7], [8]).

The q-analogue involves a new summable almost poised terminating ${}_3\phi_2$, discovered independently by Bressoud [4]. This (see (9)) is given in §2, together with other known basic hypergeometric summation theorems included for completeness. In §§3, 4 and 5 we consider the cases $p = 3$, 4, 6, respectively. In a sense to be explained in §6, these are the most interesting cases. Of the expressions we give for q-analogues, namely, M_q in Theorem 3.1, J_q in Theorem 4.1, G_q in Theorem 4.2 and L_q in Theorem 5.1, the one which specialises precisely when $q = 1$ to the correct expression (1) is given in Theorem 4.2.

For permutation problems, q-analogues of generating series often appear when a set of permutations is enumerated with respect to the inversion number or the major index (see [6]), marked by q. Since these are not class functions for the symmetric group, they cannot be used in conjunction with the group algebra of S_n to derive our result. However, the group algebra of the symmetric group is abstractly