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INDEX FOR PAIRS OF FINITE VON NEUMANN ALGEBRAS

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The Jones' index of a pair $N \subset M$ of finite von Neumann algebras with finite dimensional centers has been given two definitions: one ring-theoretic, and one using Markov traces. We extend here the second definition to the case of finite, σ-finite von Neumann algebras and we show that the two definitions agree when the algebras are direct sums of finite factors. We also study Markov traces on such pairs.

Introduction. The purpose of the present article is twofold:

(1) If $N \subset M$ is a pair of direct sums of finite factors, we prove that the index $[M : N]$ defined in Chapter 3 of [**3**] is equal to the ring-theoretic index introduced in Chapter 2 of [**3**].

(2) We give a definition of the index for a pair as above in terms of canonical objects associated to N and M such as center-valued traces and coupling operators.

In fact, we present a solution of problem (2) providing a framework in which problem (1) is easily solved. More precisely, let M be a finite, σ-finite von Neumann algebra and let N be a von Neumann subalgebra of M containing the identity of M. If N is of finite index in M, i.e. if M acts on some Hilbert space H in such a way that the commutant N'_H of N is finite, and that the coupling operators $c_M(H)^{\pm 1}$ and $c_N(H)^{\pm 1}$ are bounded, then we define two bounded, linear, normal maps

$$C_N^M \in L_*(Z(N)) \quad \text{and} \quad D_N^M \in L_*(Z(M))$$

which do not depend on the chosen representation and which have the same spectral radius.

Thus the index of N in M, denoted by $[M : N]$, is the common spectral radius of C_N^M and D_N^M.

Let $(M, L^2(M), J, P)$ be the standard form of M ([**4**]). The basic construction associated to the pair $N \subset M$ gives the finite von Neumann algebra $JN'J$, denoted by $\langle M, e_N \rangle$. Then M is of finite index in $\langle M, e_N \rangle$ and $[\langle M, e_N \rangle : M] = [M : N]$, so it is possible to