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OPERATOR-VALUED FEYNMAN INTEGRALS VIA CONDITIONAL FEYNMAN INTEGRALS

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In this paper we use the concept of the conditional Feynman integral to obtain the analytic operator-valued Feynman integral of various functions.

1. Introduction. In **[1]** Cameron and Storvick introduced a very general analytic operator-valued function space "Feynman integral", $J_q^{\text{an}}(F)$, which mapped an $L_2(\mathbb{R}^\nu)$ function ψ into an $L_2(\mathbb{R}^\nu)$ function $(J_q^{\text{an}}(F)\psi)(\vec{\xi})$. Further work involving the $L_2 \to L_2$ theory includes **[2, 3, 16–18]**. In **[4, 19]** the existence of the Feynman integral as an operator from $L_1(\mathbb{R})$ to $L_\infty(\mathbb{R})$ was studied. Finally in **[20]**, an $L_p \to L_{p'}$ theory, $1/p + 1/p' = 1$, was developed for $1 < p \leq 2$. Related stability results were established in **[10, 25]**.

In **[15]**, Chung and Skoug introduced the concept of a conditional Feynman integral. In this paper we further develop this concept and proceed to express operator-valued Feynman integrals in terms of conditional Feynman integrals. In particular we show that various operator-valued Feynman integrals can be obtained using the formula

$$(J_q^{\text{an}}(F)\psi)(\vec{\xi}) = \int_{\mathbb{R}^\nu} E^{\text{anf}_q}(F|X)(\vec{\xi})(\vec{\eta}) \left[\frac{q}{2\pi i T}\right]^{\nu/2} \cdot \exp\left\{\frac{qi}{2T}\|\vec{\eta} - \vec{\xi}\|^2\right\} \psi(\vec{\eta})\, d\vec{\eta} \tag{1.1}$$

where $E^{\text{anf}_q}(F|X)$ is the conditional analytic Feynman integral of F given X. Thus $J_q^{\text{an}}(F)$ can be interpreted as an integral operator with kernel

$$\left[\frac{q}{2\pi i T}\right]^{\nu/2} \exp\left\{\frac{qi}{2T}\|\vec{\eta} - \vec{\xi}\|^2\right\} E^{\text{anf}_q}(F|X)(\vec{\xi})(\vec{\eta}).$$

In **[5]**, Cameron and Storvick introduced a Banach algebra $S(\nu)$ of functions on Wiener space which are a kind of stochastic Fourier transform of Borel measures on $L_2^\nu[0, T]$. In §3 of this paper we show that for all F in $S(\nu)$, $J_q^{\text{an}}(F)$ is given by (1.1) and can be