

## ISOMORPHISMS AMONG MONODROMY GROUPS AND APPLICATIONS TO LATTICES IN $PU(1, 2)$

JOHN KURT SAUTER, JR.

**The discreteness of some monodromy groups in  $PU(1, 2)$  is proved. G. D. Mostow's conjecture on a necessary and sufficient condition for the discreteness of monodromy subgroups of  $PU(1, 2)$  is established. Some isomorphisms and inclusion relations among the monodromy groups are given.**

**1. Introduction.** In [DM], Deligne and Mostow define certain monodromy subgroups of  $PU(1, n)$  which are closely related to the groups Mostow studied in his earlier work [M-1]. The connection between these two is made clear in [M-2] and [M-3]. Each of the papers investigates the discreteness of the groups. Thereafter, in case  $n > 3$ , Mostow gives a necessary and sufficient condition for the groups to be discrete in  $PU(1, n)$  [M-4]. He conjectured that his condition would also hold in dimensions two and three (apart from stated exceptions). This paper considers the monodromy subgroups of  $PU(1, 2)$ . The discreteness of some monodromy groups is proved in §3. Mostow's conjecture is verified in §4. The volumes of the fundamental domains for the groups are computed in §5 and are used to find the indices for the inclusion relations among the monodromy groups given in §6. The isomorphisms given throughout this paper were discovered using computer investigations of the fundamental domains as a guide. The proofs however are completely independent of the computer work. The following brief summary of [DM], [M-1], [M-2], and [M-3] introduces notation and results needed in the remaining sections.

**2. Preliminaries. Mostow's work on discrete groups generated by complex reflections.** The following results are contained in [M-1] which arose out of Mostow's exploration of the limits of the validity in the case of  $\mathbf{R}$ -rank 1 groups of Margulis' Theorem, *Irreducible lattices in semisimple Lie groups of  $\mathbf{R}$ -rank greater than 1 are arithmetic*. Motivated by Makarov's (for  $n = 3$ ) and Vinberg's (for  $n \leq 5$ ) construction of nonarithmetic lattices in  $SO(n, 1)$  using reflections in faces of geodesic polyhedra in real hyperbolic  $n$ -space  $\mathbf{R}h^n$ , Mostow considered subgroups in the isometry group  $PU(n, 1)$  of complex