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THE C^*-ALGEBRAS GENERATED BY PAIRS OF SEMIGROUPS OF ISOMETRIES SATISFYING CERTAIN COMMUTATION RELATIONS

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Arising in the computation of the Arveson-Powers index for $*$-endomorphisms of $\mathfrak{B}(\mathfrak{H})$ is the notion of a pair of one-parameter semigroups of isometries $\mathscr{U} = \{U_t : t \in \Gamma^+\}$ and $\mathscr{S} = \{S_t : t \in \Gamma^+\}$ satisfying the commutation relations $S_t^* U_t = e^{-\lambda t} I$, for Γ the set of real numbers. If Γ is any subgroup of $\mathbb{R}$ we show that the C^*-algebra $\mathfrak{A}_\Gamma$ generated by $\mathscr{U}$ and $\mathscr{S}$ is canonically unique. $\mathfrak{A}_\gamma$ is simple if and only if Γ is dense in $\mathbb{R}$.

I. Introduction. According to the von Neumann-Wold decomposition for an isometry V acting on a Hilbert space $\mathfrak{H}$, $\mathfrak{H}$ may be decomposed into an orthogonal direct sum of reducing Hilbert subspaces $\mathfrak{H}_1$, $\mathfrak{H}_2$ for V, where $V|_{\mathfrak{H}_1}$ is a unitary operator and $V|_{\mathfrak{H}_2}$ is a pure isometry. In [**6**], L. A. Coburn characterized the C^*-algebra $C^*(V)$ generated by an isometry. If V is completely unitary then as is well known, $C^*(V)$ is isometrically $*$-isomorphic to $C(\sigma(V))$, the algebra of complex-valued continuous functions on the spectrum of V. If V has a non-trivial pure isometric part, $C^*(V)$ contains a closed two-sided ideal which is isomorphic to the compact operators $\mathscr{K}$. The quotient algebra $C^*(V)/\mathscr{K}$ is isomorphic to the algebra of continuous functions on the circle, [**6**].

Generalizations of this result (see [**4**], [**7**]–[**10**], [**12**]) made by Coburn and other authors have taken various forms. For example, the study of C^*-algebras generated by a semigroup of isometries has led to interesting developments in the theory of an index for algebras of operators. This theory is modelled on the theory of Fredholm operators in $\mathfrak{B}(\mathfrak{H})$, and has led to some interesting connections between the notions of topological and analytic index, [**8**]–[**10**].

In [**12**], R. G. Douglas analyzed the structure of the C^*-algebras $\mathfrak{A}_\Gamma$ generated by one-parameter semigroups of isometries $\mathscr{V}_\Gamma = \{V_\gamma : \gamma \in \Gamma^+\}$, where Γ is a subgroup of the real numbers. Without making any assumptions about the continuity of the mapping $\gamma \to V_\gamma$, Douglas showed that the C^*-algebra $\mathfrak{A}_\Gamma$ is canonically unique. This analysis