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ON ρ-MIXING EXCEPT ON SMALL SETS

RICHARD C. BRADLEY

For stochastic processes, some conditions of " ρ-mixing except on small sets" are shown to be equivalent to the (Rosenblatt) strong mixing condition.

I. Introduction. Suppose $(\Omega, \mathscr{F})$ is a measurable space. For any probability measure μ on $(\Omega, \mathscr{F})$, and any two σ-fields $\mathscr{A}$ and $\mathscr{B} \subset \mathscr{F}$, define the following measures of dependence:

$$\alpha(\mathscr{A}, \mathscr{B}; \mu) := \sup|\mu(A \cap B) - \mu(A)\mu(B)|, \qquad A \in \mathscr{A}, \ B \in \mathscr{B},$$

$$\phi(\mathscr{A}, \mathscr{B}; \mu) := \sup|\mu(B|A) - \mu(B)|, \qquad A \in \mathscr{A}, \ B \in \mathscr{B}, \ \mu(A) > 0,$$

$$\lambda(\mathscr{A}, \mathscr{B}; \mu) := \sup \frac{|\mu(A \cap B) - \mu(A)\mu(B)|}{[\mu(A)\mu(B)]^{1/2}}, \qquad A \in \mathscr{A}, \ B \in \mathscr{B},$$

$$\rho(\mathscr{A}, \mathscr{B}; \mu) := \sup|\mathrm{Corr}_\mu(f, g)|, \qquad f \in L^2(\Omega, \mathscr{A}, \mu), \ g \in L^2(\Omega, \mathscr{B}, \mu),$$

$$\beta(\mathscr{A}, \mathscr{B}; \mu) := \sup(1/2) \sum_{i=1}^{I} \sum_{j=1}^{J} |\mu(A_i \cap B_j) - \mu(A_i)\mu(B_j)|$$

where the last sup is taken over all pairs of partitions $\{A_1, \dots, A_I\}$ and $\{B_1, \dots, B_J\}$ of Ω such that $A_i \in \mathscr{A}$ for all i and $B_j \in \mathscr{B}$ for all j. Here of course $\mu(B|A) := \mu(A \cap B)/\mu(A)$, $0/0$ is interpreted to be 0, and

$$\mathrm{Corr}_\mu(f, g) := \frac{E_\mu(f - E_\mu f)(g - E_\mu g)}{E_\mu^{1/2}(f - E_\mu f)^2 E_\mu^{1/2}(g - E_\mu g)^2}$$

where $E_\mu h := \int_\Omega h \, d\mu$. In what follows, we shall be working with a given probability measure P, and these definitions will be used with $\mu = P$ and with $\mu = P(\cdot|D)$ for various events D.

Suppose $X := (X_k, \ k \in \mathbb{Z})$ is a strictly stationary sequence of random variables on a probability space $(\Omega, \mathscr{F}, P)$. For $-\infty \le J \le L \le \infty$ let $\mathscr{F}_J^L$ denote the σ-field of events generated by the r.v.'s