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ON THE GENERALIZED PRINCIPAL IDEAL THEOREM AND KRULL DOMAINS

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If $R \subset T$ is an integral extension of domains and R is Noetherian, then T satisfies (the conclusion of the) generalized principal ideal theorem (or GPIT for short). An example is given of a two-dimensional quasilocal domain R satisfying GPIT such that the integral closure of R is finite over R but does not satisfy GPIT. If a commutative ring R satisfies GPIT and an ideal I of R is generated by an R-sequence, then R/I satisfies GPIT. If R is a Noetherian domain and G is a torsionfree abelian group, then $R[G]$ satisfies GPIT. An example is given of a three-dimensional quasilocal Krull domain that does not satisfy GPIT because its maximal ideal is the radical of a 2-generated ideal.

1. Introduction. All rings considered in this paper are commutative with identity. Let R be a ring. As in [**4**], we say that R satisfies PIT (for "principal ideal theorem") if $ht(P) \leq 1$ for each prime ideal P of R which is minimal over a principal ideal of R. According to Krull's *Hauptidealsatz*, which has been called "the most important single theorem in the theory of Noetherian rings" by Kaplansky [**12**, p. 104], each Noetherian ring satisfies PIT. So does each Krull domain. A generalization of this fact has been noted by Davis [**5**, p. 182]. Additional examples of rings satisfying PIT were obtained in [**4**] (see especially [**4**, Corollary 3.5, Theorem 3.10, Corollary 3.11 and Theorem 6.5]). Much of [**4**] addressed the stability of "satisfies PIT" under various ring-theoretic passages. The purpose of this paper is to study similar questions concerning GPIT (for "generalized principal ideal theorem"). By definition, a ring R satisfies GPIT if $ht(P) \leq n$ for each prime ideal P of R which is minimal over an n-generated ideal of R.

According to Krull's generalized principal ideal theorem [**12**, Theorem 152] (also known as Krull's altitude theorem [**13**, Theorem 9.3]), each Noetherian ring satisfies GPIT. However, not every Krull domain satisfies GPIT. The first example of this phenomenon seems to be the one discovered by Eakin and Heinzer in 1969 after further analysis of their work in [**6**] concerning a construction of Rees [**14**]. Later