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# $L$-HARMONIC FUNCTIONS AND THE EXPONENTIAL SQUARE CLASS

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**It is proved for a restricted class of second order linear differential operators $L$ if $Lu = 0$ in $\mathbf{R}_+^{d+1}$, $u|_{\mathbf{R}^d} = f$ then if the Lusin area integral of $u$, $Su \in L^\infty$, $f$ is in the exponential square class. This extends the work of Chang, Wilson and Wolff who proved the same result for harmonic $u$ [3].**

**1. Introduction.** Let

$$L = \sum_{i,j=1}^{d+1} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial}{\partial x_j}\right)$$

be a second order differential operator in divergence form whose coefficients $a_{ij}$ are bounded and measurable functions on $\mathbf{R}_+^{d+1}$, $a_{ij} = a_{ji}$. $L$ is strictly elliptic, i.e., $\exists\lambda > 0$ such that

$$\frac{1}{\lambda}|\xi|^2 \le \sum_{i,j=1}^{d+1} \xi_i a_{ij} \xi_j \le \lambda|\xi|^2.$$

Then if $u$ is a function where $Lu = 0$ in $\mathbf{R}_+^{d+1}$, $u|_{\mathbf{R}^d} = f$, $u$ is said to be the $L$-harmonic extension of $f$. (Note: In what follows the summation convention will be used. Sums are $i, j = 1, 2, \dots, d+1$ unless otherwise indicated.)

As in the case $L = \Delta =$ the Laplacian there is a measure associated with $L$, called $L$-harmonic measure, written $d\omega$.

There has been a considerable body of work in the last 30 years on the extension of results for harmonic functions to $L$-harmonic functions. The purpose of this paper is to extend a recent result of Chang, Wilson, Wolff, to the $L$-harmonic case.

Let $u$ be a harmonic (or $L$-harmonic) function; let

$$\Gamma_\alpha(x) = \{(y, t) \in \mathbf{R}_+^{d+1} \mid |x - y| < \alpha t\}$$

be the cone in $\mathbf{R}_+^{d+1}$ over $x \in \mathbf{R}^d$ of aperture $\alpha$;

$$u^*(x) = \sup_{(y,t)\in\Gamma_\alpha(x)} |u(y, t)|$$