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TWO APPLICATIONS OF THE UNIT NORMAL BUNDLE OF A MINIMAL SURFACE IN R^N

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Dedicated to Professor Shingo Murakami on his sixtieth birthday

A Gauss parametrization of a minimal surface in R^3 is well known. We prove a generalization.

THEOREM A. *Let U be an open set of $S^N(1)$ and f a function on U such that*

$$\Delta_{S^N(1)} f = -Nf$$

and 0 is an eigenvalue of $\operatorname{Hess} f + f\langle\ ,\ \rangle$ *of multiplicity $N-2$, where $\langle\ ,\ \rangle$ is the metric of $S^N(1)$ and $\Delta_{S^N(1)}$ is the Laplacian of $S^N(1)$. Then the map of U into R^{N+1} defined by*

$$(*)\qquad f\eta + \operatorname{grad} f$$

is of rank 2 and gives a minimal surface, where η is the identity map on $S^N(1)$. Conversely, for a minimal surface M in R^{N+1}, a neighborhood of each point of M without geodesic points has this representation.

If M is a complete orientable minimal surface of finite total curvature, then there is a global representation $(*)$ of M. Using this idea, we obtain the following.

THEOREM B. *Let M be a complete orientable minimal surface of finite total curvature in R^{N+1}. Then there exist a positive real number $c(N)$ depending on N such that*

$$\operatorname{index}(M) \le c(N) \int (-K) * 1_M,$$

where K is the Gauss curvature of M and $*1_M$ is the area form of M.

Theorem B gives an answer for an open question posed by Cheng and Tysk in **[CY1]**. After this paper was submitted, the author learned that Cheng and Tysk in **[CT2]** obtained a similar result as Theorem B by using another Gauss map (generalized Gauss map).