PACIFIC JOURNAL OF MATHEMATICS
Vol. 147, No. 2, 1991

# RIGHT ORDERABLE GROUPS THAT ARE NOT LOCALLY INDICABLE

GEORGE M. BERGMAN

**The universal covering group of SL(2, R) is right orderable, but is not locally indicable; in fact, it contains nontrivial finitely generated perfect subgroups.**

**Introduction.** A group $G$ is called *right orderable* if it admits a total ordering $\leq$ such that $a \leq b \Rightarrow ac \leq bc \quad (a, b, c \in G)$. It is known that a group has such an ordering if and only if it is isomorphic to a group of order-preserving permutations of a totally ordered set [**4**, Theorem 7.1.2].

A group is called *locally indicable* if each of its finitely generated nontrivial subgroups admits a nontrivial homomorphism to $\mathbf{Z}$. Every locally indicable group is right orderable (see [**4**, Theorem 7.3.11]); it was an open question among workers in the area whether the converse was true (equivalent to [**9**, Problem 1]). This note gives a counterexample, and a modified example showing that a finitely generated right orderable group can in fact be a perfect group.

Related to the characterization of right orderable groups in terms of actions on totally ordered sets is the result that the fundamental group of a manifold $M$ is right orderable if and only if the universal covering space of $M$ can be embedded over $M$ in $M \times \mathbf{R}$. After distributing a preprint of this note, I was informed by W. Thurston and P. Kropholler that examples with the same properties were already known among topologists from this point of view (see §6 below. For another topological use of right ordered groups, in this case locally indicable ones, see [**11**].) However, as the present examples are easily established and self-contained, they seem worth presenting.

Since the classes of locally indicable groups and of right orderable groups are distinct, it will now be of interest to investigate whether various results that have been proved for the former also hold for the latter; cf. [**8**], [**2**, §9], [**3**, §4].

**2. The group** $G$. Let $G$ denote the universal covering group of $\mathrm{SL}(2, \mathbf{R})$. Elements of $G$ may be thought of as linear transformations