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RANK-2 FANO BUNDLES OVER A SMOOTH QUADRIC Q_3

IGNACIO SOLS, MICHAŁ SZUREK, AND JAROSŁAW A. WIŚNIEWSKI

In the present paper we examine rank-2 stable bundles over Q_3 with $c_1 = 0$ and $c_2 = 2$ or 4.

This paper is a continuation of [7] where rank-2 Fano bundles over $\mathbb{P}^3$ and Q_3 were studied. Let us recall that a bundle $\mathcal{E}$ is called Fano if its projectivization $\mathbb{P}(\mathcal{E})$ is a Fano manifold, i.e. a manifold with ample first Chern class $c_1(\mathbb{P}(\mathcal{E}))$. In the present paper we examine rank-2 stable bundles over Q_3 with $c_1 = 0$ and $c_2 = 2$ or 4. These are the cases whose knowledge was necessary to complete the classification of rank-2 Fano bundles over Q_3. They are very different: if $\mathcal{E}$ is stable with $c_1 = 0$, $c_2 = 2$ then its first twist $\mathcal{E}(1)$ is spanned by global sections (see Proposition 1), whereas if $c_2 = 4$ then for a general $\mathcal{E}$ from a component in the moduli $\mathcal{E}(1)$ has no section at all (Proposition 3). We complete the classification of rank-2 Fano bundles over Q_3. The results of §3 from [7] and of the present paper can be summarized in the following

THEOREM. *Let $\mathcal{E}$ be a rank-2 Fano bundle over Q_3. If $c_1\mathcal{E} = -1$ then $\mathcal{E}$ is either $\mathcal{O} \oplus \mathcal{O}(-1)$ or the spinor bundle $\underline{E}$. If $c_1\mathcal{E} = 0$ then $\mathcal{E}$ is either $\mathcal{O} \oplus \mathcal{O}$, or $\mathcal{O}(-1) \oplus \mathcal{O}(1)$, or any stable bundle with $c_2 = 2$* (*see a corollary in* §1 *for a complete description of such bundles*).

Let us recall that the spinor bundle $\underline{E}$ on an odd-dimensional quadric $Q_{2\nu+1}$ is the restriction of the universal 2^ν-bundle on the Grassmannian $\mathrm{Gr}(2^\nu, 2^{\nu+1})$. Then $\underline{E}^* = \underline{E}(1)$. On an even-dimensional quadric $Q_{2\nu}$, $\nu \geq 2$, there are two spinor bundles, corresponding to the two reguli of ν-planes. The following characterization of the bundles with no intermediate cohomology was proved in [**1**]:

THEOREM. *For a vector bundle F on Q_n, $n \geq 2$, it is $H^i(F(l)) = 0$ for all $0 < i < n$, $l \in \mathbb{Z}$, if and only if F is a direct sum of line bundles $\mathcal{O}(l)$ and of their tensor product with spinor bundles.*