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DUALITY AND INVARIANTS FOR BUTLER GROUPS

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A duality is used to develop a complete set of numerical quasi-isomorphism invariants for the class of torsion-free abelian groups consisting of strongly indecomposable cokernels of diagonal embeddings $A_1 \cap \cdots \cap A_n \to A_1 \oplus \cdots \oplus A_n$ for n-tuples $(A_1, \ldots, A_n)$ of subgroups of the additive group of rational numbers.

A major theme in the theory of abelian groups is the classification of groups by numerical invariants. For the special case of torsion-free abelian groups of finite rank, one must first consider the decidedly non-trivial problem of classification up to quasi-isomorphism. To this end, we develop a contravariant duality on the quasi-homomorphism category of T-groups for a finite distributive lattice T of types.

A *Butler group* is a finite rank torsion-free abelian group that is isomorphic to a pure subgroup of a finite direct sum of subgroups of Q, the additive group of rationals. Isomorphism classes of subgroups of Q, called *types*, form an infinite distributive lattice. For a finite distributive sublattice T of types, a *T-group* is a Butler group G with each element of the *typeset of* G (the set of types of pure rank-1 subgroups of G) in T. Each Butler group is a T-group for some T, since Butler groups have finite typesets [**BU1**], but T is not, in general, unique. There are various characterizations of Butler groups, as found in [**AR2**], [**AR3**], and [**AV1**], but a complete structure theory has yet to be determined. As E. L. Lady has pointed out in [**LA1**] and [**LA2**], the theory generalizes directly to Butler modules over Dedekind domains.

Define B_T to be the category of T-groups with morphism sets $Q \otimes_Z \mathrm{Hom}_Z(G, H)$. Isomorphism in B_T is called *quasi-isomorphism* and an indecomposable in B_T is called *strongly indecomposable.* B. Jońsson in [**JO**] showed that direct sum decompositions in B_T are unique up to order and quasi-isomorphism (see [**AR1**] for the categorical version). Thus, classification of T-groups up to quasi-isomorphism depends only on the classification of strongly indecomposable T-groups.

A complete set of numerical quasi-isomorphism invariants for strongly indecomposable T-groups of the form $G = G(A_1, \ldots, A_n)$,