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HARMONIC MAJORIZATION OF A SUBHARMONIC FUNCTION ON A CONE OR ON A CYLINDER

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To Professor N. Yanagihara on his 60th birthday

For a subharmonic function u defined on a cone or on a cylinder which is dominated on the boundary by a certain function, we generalize the classical Phragmén-Lindelöf theorem by making a harmonic majorant of u and show that if u is non-negative in addition, our harmonic majorant is the least harmonic majorant. As an application, we give a result concerning the classical Dirichlet problem on a cone or on a cylinder with an unbounded function defined on the boundary.

1. Introduction. Let $\mathbb{R}$ and $\mathbb{R}_+$ be the sets of all real numbers and all positive real numbers, respectively. The m-dimensional Euclidean space is denoted by $\mathbb{R}^m$ $(m \geq 2)$ and O denote the origin of it. By ∂S and $\overline{S}$, we denote the boundary and the closure of a set S in $\mathbb{R}^m$. Let $|P - Q|$ denote the Euclidean distance between two points P, $Q \in \mathbb{R}^m$. A point on $\mathbb{R}^m$ $(m \geq 2)$ is represented by (X, y), $X = (x_1, x_2, \dots, x_{m-1})$. We introduce the spherical coordinates (r, Θ), $\Theta = (\theta_1, \theta_2, \dots, \theta_{m-1})$, in $\mathbb{R}^m$ which are related to the coordinates (X, y) by

$$\begin{cases} x_1 = r\left(\prod_{j=1}^{m-1} \sin\theta_j\right), & y = r\cos\theta_1, \\ x_{m+1-k} = r\left(\prod_{j=1}^{k-1} \sin\theta_j\right)\cos\theta_k & (m \geq 3,\ 2 \leq k \leq m-1), \\ x_1 = r\cos\theta_1, \quad y = r\sin\theta_1 & (m = 2), \end{cases}$$

where $0 \leq r < +\infty$ and $-\frac{1}{2}\pi \leq \theta_{m-1} < \frac{3}{2}\pi$ $(m \geq 2)$, $0 \leq \theta_j \leq \pi$ $(m \geq 3,\ 1 \leq j \leq m-2)$. The unit sphere and the surface area $2\pi^{m/2}\{\Gamma(m/2)\}^{-1}$ of it are denoted by $\mathbb{S}^{m-1}$ and s_m $(m \geq 2)$, respectively. The upper half unit sphere $\{(1, \Theta) \in \mathbb{S}^{m-1};\ 0 \leq \theta_1 < \frac{\pi}{2}$ (if $m = 2$, then $0 < \theta_1 < \pi$)$\}$ is also denoted by $\mathbb{S}_+^{m-1}$ $(m \geq 2)$. For simplicity, a point $(1, \Theta)$ on $\mathbb{S}^{m-1}$ and a set S, $S \subset \mathbb{S}^{m-1}$, are often identified with Θ and $\{\Theta; (1, \Theta) \in S\}$, respectively. For two