PACIFIC JOURNAL OF MATHEMATICS
Vol. 148, No. 2, 1991

THE p-PARTS OF BRAUER CHARACTER DEGREES IN p-SOLVABLE GROUPS

YOU-QIANG WANG

Let G be a finite group. Fix a prime integer p and let e be the largest integer such that p^e divides the degree of some irreducible Brauer character of G with respect to the same prime p. The primary object of this paper is to obtain information about the structure of Sylow p-subgroups of a finite p-solvable group G in knowledge of e.

As applications, we obtain a bound for the derived length of the factor group of a solvable group G relative to its unique maximal normal p-subgroup in terms of the arithmetic structure of its Brauer character degrees and a bound for the derived length of the factor group of G relative to its Fitting subgroup in terms of the maximal integer e when p runs through the prime divisors of the order of G.

All groups considered are finite. Let G be a group and p be a prime. We denote by $\mathrm{IBr}_p(G)$ the set of irreducible Brauer characters of G with respect to the prime p. For the same prime p, let $e_p(G)$ be the largest integer e such that p^e divides $\varphi(1)$ for some $\varphi \in \mathrm{IBr}_p(G)$. Let P be a Sylow p-subgroup of G. Then the Sylow p-invariants of G are defined as follows:

(1) $b_p(G)$, where $p^{b_p(G)}$ is the order of P;

(2) $c_p(G)$, the class of P, that is, the length of the (upper or) lower central series of P;

(3) $dl_p(G)$, the length of the derived series of P;

(4) $ex_p(G)$, where $p^{ex_p(G)}$ is the exponent of P, that is, the greatest order of any element of P.

For a p-solvable group G, we let $l_p(G)$ and $r_p(G)$ denote the p-length and p-rank (respectively) of G, i.e. $r_p(G)$ is the largest integer r such that p^r is the order of a p-chief factor of G.

We give a linear bound for $r_p(G/O_p(G))$ and a logarithmic bound for $l_p(G/O_p(G))$ in terms of $e_p(G)$. Then, using induction on $l_p(G/O_p(G))$, we obtain bounds for $c_p(G/O_p(G))$, $dl_p(G/O_p(G))$ and $ex_p(G/O_p(G))$ in terms of $e_p(G)$.