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ORDERED GROUPS AND CROSSED PRODUCTS OF C^*-ALGEBRAS

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We define and analyse the concept of a crossed product of a C^*-algebra A by a semigroup. For a large class of semigroups we show that the crossed product is primitive if A is, and our constructions also give rise to simple C^*-algebras. Conditions are given for when the crossed product is type I or nuclear, and when covariant representations of a C^*-dynamical system give rise to faithful and/or irreducible representations of the crossed product.

Introduction. The theory of crossed products of C^*-algebras by automorphism groups is a deep and interesting area of the modern theory of operator algebras, as well as being a rich source of examples. It is natural to try to extend the ideas of this area to a more general setting. One way to do this is to consider crossed products by semigroups, and this paper develops some aspects of the theory. Surprisingly (or perhaps not) if the semigroup does not look much like a group the results turn out to be radically different in many respects from the classical case. For instance, the group C^*-algebra of an abelian group is of course itself abelian, and so, from the point of view of C^*-theory, not very interesting. But for a large and natural class of semigroups (namely the positive cones of abelian ordered groups) their C^*-algebras are not only non-abelian but actually primitive. This is useful because the primitive (and the simple) C^*-algebras are in a sense the building blocks of C^*-theory.

If we come down to very concrete detail, and look at the additive semigroup $\mathbf{N}$ of natural numbers, we find that its C^*-algebra is the Toeplitz algebra, i.e. the C^*-algebra generated by all Toeplitz operators with continuous symbol on the unit circle. Indeed, for any cone as above, its C^*-algebra can be faithfully represented as a C^*-algebra of Toeplitz operators in a generalized sense (see [12]). For the related situation of C^*-algebras generated by multivariable Wiener-Hopf operators see [11] and [15]. The papers [4], [10] and [21] are also relevant. Indeed a very interesting theory of crossed products by semigroups is developed in [10]. This theory is quite different from ours however as the crossed product in [10] is in general a non-self-adjoint algebra. If (A, α, G) is a separable C^*-dynamical system with