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GENERALIZED CLIFFORD-LITTLEWOOD-ECKMANN GROUPS II: LINEAR REPRESENTATIONS AND APPLICATIONS

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This paper presents applications of the decomposition of generalized Clifford-Littlewood-Eckmann groups, or CLE-groups, which are given by presentations of the type

$$\mathbf{G} = \langle \omega, a_1, \dots, a_r | \omega^n = 1,\ a_i^n = \omega^{e(i)}\ \forall i,\ a_i a_j = \omega a_j a_i\ \forall i < j,\ \omega a_i = a_i \omega\ \forall i \rangle.$$

We begin by studying the irreducible complex representations of the "building block groups" of orders n^2 and n^3, and how the representations for the composite groups are constructed from them. This of course also gives a complete set of inequivalent irreducible matrix representations for the generalized Clifford algebras corresponding to these groups. We apply these representation-theoretic results to determine the size of the maximal abelian subgroups of these groups, and to present a generalization of a result of Littlewood on maximal sets of anticommuting matrices. In the final section we consider an alternative generalization of the CLE-groups, in which we require $a_i^n = 1$, but allow $a_i a_j = \omega^k a_j a_i$ for fixed k dividing n, where possibly $k > 1$. The irreducible complex representations of these groups are then calculated.

Introduction. These representations have been studied from the standpoint of projective representations of $(\mathbb{Z}/n\mathbb{Z})^r$ in **[S-I]**. However we feel that the presentation given here is somewhat clearer. The results again are of interest to physicists in a number of applications (see **[S-I]**, **[Kw]**). Throughout this paper the notation and conventions will follow those of **[Sm1]**, **[Sm2]**. The results of **[Sm2]** concerning the explicit decomposition of the groups will be used extensively here. The corresponding generalized Clifford algebras are studied in **[Sm3]**.

1. Linear representations. The first application of the decomposition results obtained in **[Sm2]** is the determination of the irreducible complex representations for the generalized group $\mathbf{G} = \langle \omega, a_1, \dots, a_r | \omega^n = 1,\ a_i^n = \omega^{e(i)}\ \forall i,\ a_i a_j = \omega a_j a_i\ \forall i < j,\ \omega a_i = a_i \omega\ \forall i \rangle$. To begin we need some general facts about representations of finite groups, which are found in **[I]**.