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CONJUGATES OF EQUIVARIANT HOLOMORPHIC MAPS OF SYMMETRIC DOMAINS

MIN HO LEE

In this paper we construct the conjugates of equivariant holomorphic maps of symmetric domains associated to morphisms of arithmetic varieties. We also prove that the conjugate of a Kuga fiber variety is another Kuga fiber variety.

0. Introduction. Let $\mathbf{G}$ be a simply connected semisimple algebraic group over $\mathbf{Q}$ that does not contain direct factors defined over $\mathbf{Q}$ and compact over $\mathbf{R}$, and let K be a maximal compact subgroup of the semisimple Lie group $G = \mathbf{G}(\mathbf{R})$. We assume that the symmetric space $D = G/K$ has a complex structure. Let Γ be a torsion free arithmetic subgroup of $\mathbf{G}$ and let $X = \Gamma \setminus D$ be the corresponding arithmetic variety. For each $\sigma \in \mathrm{Aut}(X)$ it is known (cf. [**5**], [**6**], [**7**], [**10**]) that the conjugate X^σ of X is also an arithmetic variety.

Let $\mathbf{G}'$ be another semisimple algebraic $\mathbf{Q}$-group, and consider the corresponding objects G', K', D', Γ' and X' as in the case of $\mathbf{G}$. Let $\rho: G \to G'$ be a homomorphism of Lie groups and $\tau: D \to D'$ a holomorphic map such that (ρ, τ) is an equivariant pair and $\rho(\Gamma) \subset \Gamma'$. Then τ induces the morphism $\phi: X \to X'$ of arithmetic varieties. Let D^σ and D'^σ be the universal covering spaces of X^σ and X'^σ respectively, and let $\tau^\sigma: D^\sigma \to D'^\sigma$ be the lifting of $\phi^\sigma: X^\sigma \to X'^\sigma$. Let G_0 and G'_0 be the connected components of the identity of $\mathrm{Aut}(D^\sigma)$ and $\mathrm{Aut}(D'^\sigma)$ respectively. If $\Gamma^\sigma \subset G_0$ and $\Gamma'^\sigma \subset G'_0$ are the fundamental groups of X^σ and X'^σ respectively, then we have the following result, Theorem 5.2 of this paper.

THEOREM. *There exist a finite covering G_1^σ of G_0^σ and a homomorphism $\rho_1^\sigma: G_1^\sigma \to G_0'^\sigma$ such that ρ_1^σ and τ^σ are equivariant and $\rho_1^\sigma(\Gamma^\sigma)$ is contained in Γ'^σ.*

As an application of this result we consider the conjugates of Kuga fiber varieties. Let $\mathbf{G}' = \mathrm{Sp}(V, \beta)$ for some $\mathbf{Q}$-vector space V and a nondegenerate alternating bilinear form β, and assume that $X = \Gamma\backslash D$ is compact. Then from the equivariant pair (ρ, τ) we can construct a