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CLASSIFICATION OF ESSENTIAL COMMUTANTS OF ABELIAN VON NEUMANN ALGEBRAS

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The main purpose of this paper is to classify the C^*-algebras of the form $\mathfrak{A}' + \mathscr{K}$, where $\mathfrak{A}'$ denotes the commutant of an abelian von Neumann algebra $\mathfrak{A}$, and $\mathscr{K}$ is the set of compact operators. By the famous result of Johnson and Parrott, $\mathfrak{A}' + \mathscr{K}$ is the same as the essential commutant of $\mathfrak{A}$. These algebras were studied by Plastiras in the special case in which $\mathfrak{A}$ is generated by its minimal projections and in addition all of these projections are finite dimensional. Using a theorem of Andersen, we are able to generalize Plastiras' main results to general abelian von Neumann algebras. We also study the automorphism groups and derivations of these algebras.

If $\mathfrak{A}$ is an abelian von Neumann algebra, then its projection lattice $\mathscr{L}$ is a complemented commutative subspace lattice, and of course $\mathfrak{A}' = \mathscr{L}'$. Since our results are given in terms of the lattice, we simply start with such a lattice $\mathscr{L}$ and consider the algebra $\mathscr{L}' + \mathscr{K}$. In Corollaries 6 and 9, we give necessary and sufficient conditions for two such algebras $\mathscr{L}' + \mathscr{K}$ and $\mathscr{M}' + \mathscr{K}$ to be equal or isomorphic. We then turn to automorphisms, and first catagorize those algebras for which every unitary operator implementing an automorphism splits (Theorem 11), and then determine those algebras for which every automorphism is inner (Theorem 12). These four results generalize the most important results in **[P]**. We next calculate the outer automorphism group (Corollary 20), and finally show that every derivation of such an algebra is inner (Theorem 22). This latter portion of the paper was motivated primarily by similar studies with nests, namely **[W1]**, **[W2]**, and **[DW]**.

All Hilbert spaces in this paper will be separable and infinite dimensional, and will usually be denoted by $\mathscr{H}$. $\mathscr{B}(\mathscr{H})$ will be used to denote the set of bounded operators on $\mathscr{H}$, and the set of compact operators will be given by $\mathscr{K}(\mathscr{H})$, or just $\mathscr{K}$ if the Hilbert space is clear from the context. If $\mathscr{S} \subseteq \mathscr{B}(\mathscr{H})$, then $\mathscr{S}' = \{T \in \mathscr{B}(\mathscr{H}) : TS = ST \text{ for all } S \in \mathscr{S}\}$ is the *commutant* of $\mathscr{S}$, and $\mathscr{S}''$ denotes the *double commutant* $(\mathscr{S}')'$ of $\mathscr{S}$. The *essential commutant* of $\mathscr{S}$ is $\{T \in \mathscr{B}(\mathscr{H}) : TS - ST \in \mathscr{K} \text{ for all } S \in \mathscr{S}\}$. All projections on Hilbert space will be self-adjoint.